Quiz 2

1. (5 points) Let A and B be sets. Prove: $A \subseteq B$ if and only if $A \cap B = A$. *(Please structure the proof in the outline format I demonstrated in class. Hint: start by outlining the "if and only if" and set equality proofs. Every step you fill in is more partial credit.)*

2. (5 points) Suppose $f : A \to B$ is a function with the property that $|f(S)| = |S|$ whenever $S \subseteq A$ is a finite set. Prove that f is one-to-one (injective). *(Recall that $f(S)$ means $\{f(x) \mid x \in S\}$.)*

(Fun fact #1) The set $\{f \mid f : \mathbb{N} \to \mathbb{N} \text{ and for all } n \in \mathbb{N}, \ f(n) \leq f(n+1)\}$ is uncountable.

(Fun fact #2) The set $\{f \mid f : \mathbb{N} \to \mathbb{N} \text{ and for all } n \in \mathbb{N}, \ f(n) \geq f(n+1)\}$ is countable.

(For fun) Why?