Quiz 3

1. (5 points) A subspace W of $\mathbb{R}^3$ is spanned by $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$. What is $\dim W$?

2. (5 points) A basis for $\mathbb{P}_2$ is $\mathcal{B} = \begin{pmatrix} 1 & 1+x & x^2 \end{pmatrix}$. (a) Find coordinates for $p_1(x) = 2+x+x^2$, $p_2(x) = -x + x^2$, and $p_3(x) = -1 + x + x^2$ with respect to $\mathcal{B}$. (b) Use the coordinates to determine whether these polynomials are linearly independent.

3. (1 point) A 3×3 matrix A satisfies $A^2 = 0$. What are the possible dimensions for $\operatorname{Col} A$ and $\operatorname{Nul} A$? Give them as pairs $(\dim \operatorname{Col} A, \dim \operatorname{Nul} A)$.