MATH 54 LECTURE 3. MIDTERM EXAM
JULY 22, 2016, 50 MINUTES
(6 PAGES)

Problem Number	1	2	3	4	5	Total
Score						

YOUR NAME: ______________________________

No calculators, no references except for one 8.5×11 *sheet of notes.*
Answers without justification will receive no credit.

1. (15 points) For each of the following matrices A, compute bases for $\operatorname{Col} A$, $\operatorname{Nul} A$, and $\operatorname{Row} A$.

(a) $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$ (b) $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix}$ (c) $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 4 \\ 1 & 2 & 5 \end{pmatrix}$

2. (15 points) $\mathbb{R}^{m\times n}$ is the vector space of $m \times n$ matrices with the usual addition and scalar multiplication.

(a) Let W be the set of all 2×2 matrices A where $A\vec{x} = \left(\begin{smallmatrix}1\\2\end{smallmatrix}\right)$ has at least one solution. Determine whether or not W is a subspace of $\mathbb{R}^{2\times 2}$, and if it is, give its dimension.

(b) Let U be the set of all 3×3 matrices A satisfying $A^T = -A$. Determine whether or not U is a subspace of $\mathbb{R}^{3\times 3}$, and if it is, give its dimension.

3. (15 points) Let $A = \begin{pmatrix} 5 & -6 & 0 \\ 3 & -4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Its characteristic polynomial is $p_A(\lambda) = (1+\lambda)(1-\lambda)(2-\lambda)$, which you may use if you demonstrate how to compute it.

(a) Find all $c \in \mathbb{R}$ so that the matrix $A - c^2 I_3$ is **not** invertible.

(b) Compute $A^{22} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ by first diagonalizing A.

4. (15 points) Let W be the subspace of $\mathbb{R}^3$ spanned by $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$.

   (a) Compute $\mathrm{proj}_W \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

   (b) Compute the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(\vec{x}) = \mathrm{proj}_W \vec{x}$.

   (c) Compute the dimension of $W^\perp$.

5. (15 points) $\mathbb{P}_2$ is the vector space of polynomials whose degree is at most 2. Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be defined by $T(p(x)) = p(x) - p(-x)$ for $p(x) \in \mathbb{P}_2$. For instance, $T(x+1) = (x+1) - (-x+1) = 2x$.

(a) Show that T is a linear transformation.

(b) Compute bases for the kernel of T and for the image of T. (*Range* is a synonym for *image*.)

(c) Show that T is neither one-to-one nor onto.