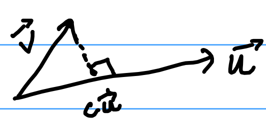
Discussion – July 14

1. Find the set of vectors orthogonal to $\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\2\\3\end{pmatrix}$.
2. Find the set of vectors orthogonal to $\begin{pmatrix}1\\2\\3\end{pmatrix}$.
3. Diagonalize $\begin{pmatrix}2&1&1\\1&2&1\\1&1&2\end{pmatrix}$. Can you make it so the P matrix has orthogonal columns?
4. Normalize $\begin{pmatrix}3\\4\\5\end{pmatrix}$
5. Find the distance between $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}2\\3\end{bmatrix}$.
6. $\mathcal{C}([0,1]) = \{$continuous functions $[0,1] \to \mathbb{R}\}$.
   $\langle f,g\rangle = \int_0^1 f(t)\,g(t)\,dt$ is an inner product on $\mathcal{C}([0,1])$
   (a) $\|\sin \pi x\|$ (b) $\operatorname{dist}(1+x,\ e^x)$
   (c) $\langle \sin \pi x, \cos \pi x\rangle$ (d) $\langle \sin \pi x, \sin 2\pi x\rangle$ $\Big\}$ orthogonal?

   Interesting but a lot of work: Compute $s_n = \langle x-\frac{1}{2}, \sin(2\pi n x)\rangle$ and $c_n = \langle x - \frac{1}{2}, \cos(2\pi n x)\rangle$ for $0 \le n \le 4$ or 5.
   Graph $y = \sum_n \left(s_n \sin(2\pi n x) + c_n \cos(2\pi n x)\right)$ and compare to $y = x - \frac{1}{2}$.

   (e) find a quadratic polynomial orthogonal to x.
7. $\vec{u} = \begin{bmatrix}1\\0\\-2\end{bmatrix}$ $\vec{v} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\vec{w} = \begin{bmatrix}0\\2\\-1\end{bmatrix}$

   (a) Compare $\|\vec{u}+\vec{v}\|$ and $\|\vec{u}\| + \|\vec{v}\|$.
   (b) Compare $|\vec{u}\cdot\vec{w}|$ and $\|\vec{u}\|\,\|\vec{w}\|$
   (c) Compute $\vec{u}^T\vec{v}$, $\vec{v}^T\vec{u}$, $\vec{u}\cdot\vec{v}$.
   (d) Compute $\vec{u}\,\vec{v}^T$. This <u>does</u> make sense.
8. If A has linearly independent columns, is A^TA invertible?
9. If $n\times n$ A has columns which sum to 1, show 1 is an eigenvalue.
10. Let $\vec{u}, \vec{v} \in \mathbb{R}^n$, $\vec{u} \neq \vec{0}$. Find a formula which gives the nearest vector to $\vec{v}$ in $\operatorname{Span}\{\vec{u}\}$.

   Hint: 

   $c\vec{u} = \operatorname{proj}_{\vec{u}} \vec{v}$ (the "projection")