Discussion – July 13

1. Diagonalize $A = \begin{pmatrix} -10 & -18 \\ 6 & 11 \end{pmatrix}$.
2. For $A = PDP^{-1}$ with $P = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & \\ & 3 \end{pmatrix}$, give (at least) two different diagonalizations of A.
3. Diagonalize $A = \begin{pmatrix} 5/3 & 0 & 2/3 \\ 0 & 3 & 0 \\ 4/3 & 0 & 7/3 \end{pmatrix}$

4. Which of the following are diagonalizable?

   (i) $A = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 2 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (iii) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

   (iv) $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ (v) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

5. Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$. Compute A^{10}.

   (Alternative method: $10 = 2^3 + 2^1$, so $A^{10} = A^{2^3} A^2$. $A^{2^3} = ((A^2)^2)^2$, so you only need four matrix multiplications to get A^{10}.)

6. Suppose A is $n \times n$ with characteristic polynomial $\lambda^n = 0$. Is $I - A$ invertible? (Show $I - A$ does not have 0 as an eigenvalue.)

7. Every year, 30% of Coke drinkers become Pepsi drinkers, and 20% of Pepsi, Coke. The remainder remain true to their brand (for the time being). As the years go on, what do their market shares converge to? (Hint: state vector $\begin{bmatrix} \text{coke} \\ \text{pepsi} \end{bmatrix}$, make matrix A transferring state from year to next, $\lim_{n \to \infty} A^n$.)