

Discussion - July 7

1. Compute dimensions:

(a) \mathbb{P}_5 (b) $\{ \vec{x} \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 + x_4 = 0 \}$

(c) $\text{col } A$ where $n \times n$ A is invertible

(d) $\text{col } A$ where 3×3 A has $\text{Nul } A = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

(e) $\text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \right\}$

(f) $\text{im } T$ and $\text{ker } T$ of $T: \mathbb{P}_3 \rightarrow \mathbb{P}_3$, $T(p(x)) = p(x) - xp'(x)$

(g) $\left\{ A \in M_{2 \times 2} \mid \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} A = A \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}$

(h) $\text{Nul} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 5 \end{pmatrix}$

2. Let $\mathcal{B} = (1 \ x \ \frac{1}{2}(3x^2 - 1))$. Find coordinates for

(a) 1 (b) x (c) x^2 (d) $\frac{1}{2}(x^2 + x)$ (e) $x^2 - x - 1$

3. Let $\mathcal{B} = (\sin x \ \cos x)$. Find coordinates for $\sin(x - \frac{\pi}{4})$

4. Let $\mathcal{B} = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$. Find coordinates for $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
Draw a picture illustrating this.

5. If $\dim V = n$, what can you say about m vectors if $m > n$? if $m < n$?