Discussion — July 6

1. Find a basis for each of the following:

(a) $\{ \vec{x} \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 + x_4 = 0 \}$

(b) $\mathrm{Col}\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$ (c) $\mathrm{Nul}\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$

(d) $\ker T$ where $T: \mathbb{P}_2 \to \mathbb{R}$ defined by $p(x) \mapsto p(2)$ (evaluation at 2)

(e) $\mathrm{im}\, T$ where $T: \mathbb{P}_2 \to \mathbb{P}_3$, $p(x) \mapsto (x-2)p(x)$

(f) $\mathrm{Span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \right\}$

(g) $\left\{ A \in M_{2\times 2} \mid \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$

(h) im and ker of $T: \mathbb{P}_3 \to \mathbb{P}_3$, $p(x) \mapsto p(x) - p'(x)$

(i) $\left\{ A \in M_{2\times 2} \mid \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} A = A \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \right\}$

2. For $A \in M_{2\times 2}$ (say $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$), show $\{I, A, A^2\}$ are linearly dependent.

3. Show that $\{ (x-2)(x-3),\ (x-1)(x-3),\ (x-1)(x-2) \}$ form a basis of $\mathbb{P}_2$. (a) Write $\frac{1}{2}x(x+1)$ as a linear combination of these three polynomials.
(b) Use this basis to come up with an inverse function for $T: \mathbb{P}_2 \to \mathbb{R}^3$ def. by $p(x) \mapsto (p(1), p(2), p(3))$.

A is $m \times n$ matrix

$\mathrm{Nul}(A)$ is solutions to
$A\vec{x} = \vec{0}$ $(\vec{x} \in \mathbb{R}^n)$

$\mathrm{Col}(A)$ is $\vec{b} \in \mathbb{R}^m$ where
$A\vec{x} = \vec{b}$ is consistent

$T : V \to W$

$\ker T$ is solutions to
$T(\vec{x}) = \vec{0}$ $(\vec{x} \in V)$

$\mathrm{im}\, T$ is $\vec{b} \in W$ where
$T(\vec{x}) = \vec{b}$ solvable

for $T : \mathbb{R}^n \to \mathbb{R}^m$, $\vec{x} \mapsto A\vec{x}$

$\mathrm{Nul}\, A = \ker T$
$\mathrm{Col}\, A = \mathrm{im}\, T$

or

$\mathrm{Nul}[T] = \ker T$
$\mathrm{Col}\, [T] = \mathrm{im}\, T$