Discussion - June 30

1. Show $\det(AB) = \det(BA)$
2. Show $\det(P^{-1}BP) = \det(B)$ when P is invertible.
3. Show $\det(P)^{-1} = \det(P^{-1})$ when P invertible.
4. Show $\det(\text{zero matrix}) = 0$.
5. If A^3 is the zero matrix, show that $\det(I - A) \neq 0$.

6. If $A^TA = I_n$, show that $\det(A) = \pm 1$.

7. Compute elementary matrices for (3×3)
   (a) $R_1 \to 6R_1$ (b) $R_2 + 3R_1 \to R_2$
   (c) $R_1 - 2R_3 \to R_1$ (d) $R_1 \leftrightarrow R_2$ (e) $R_1 \leftrightarrow R_3$
   What are their inverses?

8. What is the matrix of the disallowed operation $R_1 + 2R_2 \to R_3$? Is it invertible?

9. For which λ is $\begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$ not invertible?

10. Graph (x,y) for which $\begin{pmatrix} 2 & x \\ 1 & y \end{pmatrix}$ is not invertible.

11. Graph (x,y) for which $\begin{pmatrix} x & 2 \\ 1 & y \end{pmatrix}$ is not invertible.

12. For which x is $\begin{pmatrix} x & -1 \\ 1 & x \end{pmatrix}$ invertible?

13. Solve $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \\ 1 & 4 & 9 & 6 \end{array}\right]$ using Cramer's rule.

14. What is x_1 if $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\vec{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$? (Using Cramer's rule)