Discussion – June 29

1. Is $\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$ invertible?

2. Is $\begin{pmatrix} 1 & 2 \end{pmatrix}$ invertible?

3. Is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix}$ invertible?

4. (a) If $n \times m$ B has independent columns and $n \times n$ A is invertible, does AB have independent columns?
   (b) $m \times m$ C is invertible. Does BC have indep. cols?

5. If columns of $n \times m$ B span $\mathbb{R}^n$, A, C same as in 4,
   (a) do columns of AB span $\mathbb{R}^n$?
   (b) do columns of BC span $\mathbb{R}^n$?

6. Determine whether $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ is invertible using determinants.

7. Compute the determinant of A using cofactor expansion

   (a) $A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 4 & 1 \\ 0 & 1 & 2 & 2 \\ 3 & 0 & -1 & 7 \end{pmatrix}$

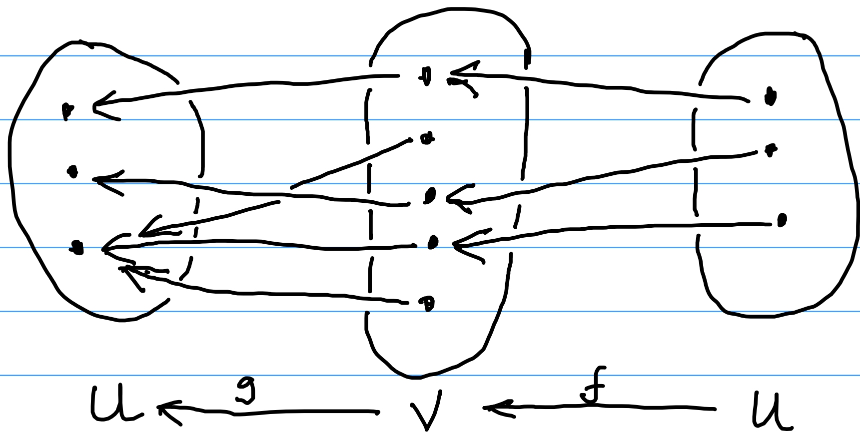
   (c) $A = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

8. Compute these determinants using row operation rules.

9. Compute $\det\begin{pmatrix} a & * & * \\ 0 & b & * \\ 0 & 0 & c \end{pmatrix}$ where stars can be anything.

10. Work out formula for $\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ using rref and/or cofactor expansion.

example of left & right inverse for finite sets:



so $f: U \to V$ and $g(f(x)) = x$
$g: V \to U$

that is, $g \circ f = id$
(identity function)

notice: f is one-to-one, g onto
f is undoable, g is get-to-able.

g is left inverse to f

f is right inverse to g