Discussion – June 27

One-to-one and Onto

1. For the following functions $f: \mathbb{R} \to \mathbb{R}$, sketch a graph, determine whether f is one-to-one, whether f is onto, and whether f is linear.

(a) $f(x) = 0$ (b) $f(x) = e^x$ (c) $f(x) = x^2$
(d) $f(x) = 2x$ (e) $f(x) = 2x + 1$. (f) $f(x) = x^3 - x$

2. A preimage of $\vec{b}$ is an $\vec{x}$ such that $\vec{b}$ is the image of $\vec{x}$ (i.e., $\vec{b} = T(\vec{x})$). Find all preimages, and determine whether the transformation is one-to-one.

(a) $T(\vec{x}) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \vec{x}$ $\quad \vec{b} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$

(b) $T(\vec{x}) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \vec{x}$ $\quad \vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(c) $T(\vec{x}) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \vec{x}$ $\quad \vec{b} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$

(d) $T(\vec{x}) = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \vec{x}$ $\quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

Matrix multiplication

1. Multiply (a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\begin{pmatrix} 4 & 5 & 6 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 8 & 6 & 2 \\ 9 & 6 & 21 \\ 3 & 2 & 7 \end{pmatrix}$ (f) $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^k$ for $k = 1, 2, 3, \ldots$ (until you get tired or recognize what is happening)

2. A is $n \times m$ B is $m \times p$

(a) Does AB have a pivot in every column if B doesn't?
(b) Does AB have a pivot in every row if A doesn't?
(c) Find B without a pivot in every row where AB does.
(same for A and columns)