Discussion — June 23

Existence

1. For A an $m \times n$ matrix, does $A\vec{x} = \vec{b}$ always have a solution, for all $\vec{b} \in \mathbb{R}^m$? Give (counter)examples.
2. For A an $m \times n$ matrix, can $A\vec{x} = \vec{b}$ always have a solution, for all $\vec{b} \in \mathbb{R}^m$, in the following scenarios?
   (a) $m > n$ (b) $m < n$ (c) $m = n$
   (give example of A if possible, else explain why not.)
3. Do $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}7\\8\\9\end{bmatrix}$ span $\mathbb{R}^3$?
4. What are all possible sets of pivot positions for a 3×5 matrix A whose columns span $\mathbb{R}^3$? Counting is OK if you don't want to list them.

Homogeneous systems

1. For an $m \times n$ matrix A, does $A\vec{x} = \vec{0}$ have a nontrivial solution in the following scenarios?
   (a) $m < n$ (b) $m > n$ (c) $m = n$
   Give examples of A where it's true and not true. If not possible to give an example, explain why.
2. What are all possible sets of pivot positions in a 5×3 matrix A where $A\vec{x} = \vec{0}$
   (a) has no nontrivial solutions (b) has nontrivial solutions
3. Give the solution set to
$$\begin{bmatrix}1 & 2 & 8 & 14\\ 2 & 0 & 4 & 8\\ 2 & -1 & 1 & 3\end{bmatrix}\vec{x} = \vec{0}$$
   in parametric vector form and as a span.
4. Do the same for the intersection of $x + y + z = 1$ and $x - y = 0$. Make sure it's a line.

Independence

1. For the following, give a dependence or demonstrate independence:

   (a) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$

2. A matrix A has a pivot in every column. Are its columns independent?

3. A square matrix has independent columns. Does it have a pivot in every row?