Discussion - June 21

1. For the following systems, solve using row reduction. Then, describe the intersection geometrically (see if you can draw it). Check your answer.

$$\begin{cases} x + 3y - z = 1 \\ 3x + 4y - 4z = 7 \\ 3x + 6y + 2z = -3 \end{cases} \qquad \begin{cases} x + y - 3z = -5 \\ -5x - 2y + 3z = 7 \\ 3x + y - z = -3 \end{cases}$$

2. a, b, c, d, e, f are constants. Solve by row reduction

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

If you must divide by a, b, c, or d, remember they might be zero, so you technically have to deal with the case it is zero and the case it isn't. What happens if $ad - bc = 0$? (Answer in terms of consistency and uniqueness.)

3. λ (the Greek letter lambda) is a constant. When does the system $\begin{cases} (\lambda - 3)x + y = 0 \\ x + (\lambda - 3)y = 0 \end{cases}$ have more than one solution? Describe them parametrically.

4. Come up with a 3×4 augmented matrix for each situation:
   i) an inconsistent system
   ii) a consistent system with the unique solution $(2, 3, 4)$
   iii) a consistent system with one free variable
   iv) a consistent system with two free variables.