Syllabus for Quiz 2

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The second quiz is in section on Tuesday, April 12. (DSP students will take it in the professor's office between 9am and 5pm. Please send me an e-mail to make arrangements.)

1 Linear algebra

- Section 6.5
  - Least-squares solutions.
  - Projection onto the column space of A (i.e., least squares solution $\hat{x}$ to $A\vec{x} = \vec{b}$ gives $A\hat{x} = \operatorname{proj}_{\operatorname{Col} A} \vec{b}$).
  - Invertibility of $A^T A$ is equivalent to uniqueness of least-squares solutions is equivalent to independence of columns of A.
  - How to compute least squares solutions given a QR factorization (I will not ask you to compute a QR factorization).
- Section 7.1
  - If $A = PDP^{-1}$ with P an orthogonal matrix, then A is symmetric.
  - Spectral theorem: if A is a symmetric matrix, then it has n real eigenvalues (possibly with multiplicity), the dimension of each eigenspace equals the multiplicity, the eigenspaces are mutually orthogonal, and A is diagonalizable as PDP^{-1} with P an orthogonal matrix.
  - Construct a symmetric matrix which a given set of eigenvalues.
  - Construct a projection matrix given an orthogonal basis, using the idea of spectral decomposition.

2 Differential equations

- Section 4.2
  - Solving second-order homogeneous linear differtial equations with real roots, distinct or not.

- The characteristic equation (also known as auxiliary equation, but we won't use that terminology).
- (Solving higher-order homogenous linear differential equations.)
- Boundary value problems. That is, solving for the constants given conditions on the solution. Initial conditions or otherwise.

- Section 4.3
  - Solving such equations when there are complex roots.
  - I would like to take a moment to point out that complex roots aren't special. When $\lambda, \overline{\lambda}$ are a pair of complex conjugate roots, then you have the solutions $C_1 e^{\lambda t} + C_2 e^{\overline{\lambda} t}$, with C_1 and C_2 possibly complex. If you choose C_1, C_2 appropriately, then you can ensure your solution is real, which is what the book does for you with $c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t)$.
  - The relationship between the real part of λ and damping. The relationship between the imaginary part of β and oscillation frequency.

- Section 4.4
  - Solving second-order linear differential equations by the method of undetermined coefficients (i.e., "guess and check"). Basically, for $ay'' + by' + cy = f(t)$, when $f(t)$ looks like the solution to some second-order homogenous linear differential equation, list out the associated eigenvalues, and guess the general version of it for y. For instance, for $f(t) = e^{\lambda t}$, guess $ce^{\lambda t}$. When λ is complex, also guess its conjugate, which implies when $f(t) = \cos \beta t$, guess $c_1 \cos \beta t + c_2 \sin \beta t$. Though, if any of the apparent eigenvalues are eigenvalues for $ay'' + by' + cy$, guess $te^{\lambda t}$, too, with powers of t up to the multiplicity.
  - Page 422 explains this again.
  - "Keep in mind that the method of undetermined coefficients applies only to nonhomogeneities that are polynomials, exponentials, sines or cosines, or products of these functions."
  - I might ask you to compute the magnitude of an oscillation.