MATH 54 QUIZ I, KYLE MILLER
MARCH 1, 2016, 40 MINUTES
(5 PAGES)

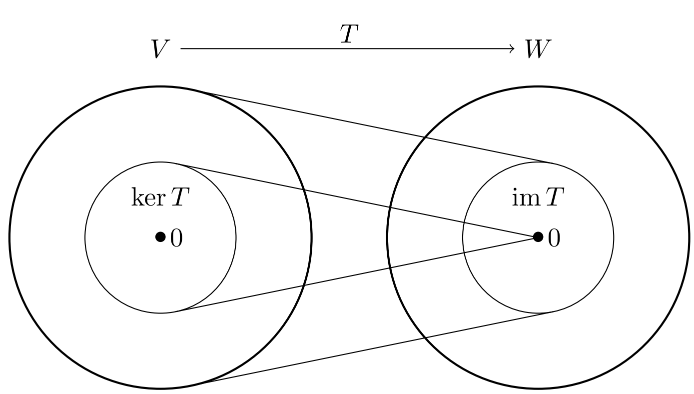
Problem Number	1	2	3	4	Total
Score					

YOUR NAME: ______________________________

No calculators, no references, no cheat sheets.
Answers without justification will receive no credit.

Glossary

$\ker T$**:** the *kernel* of a linear transformation T.
$\operatorname{im} T$**:** the *image* or *range* of a linear transformation T.
onto**:** for $T : V \to W$, $\operatorname{im} T = W$.
one-to-one**:** for $T : V \to W$, $\ker T = \{0\}$.
basis**:** a linearly independent spanning set.
dimension**:** the number of vectors in a basis for a vector space.



1. (6 points) For each of the following, find all values of $a \in \mathbb{R}$ (if any) so that the given set of vectors spans $\mathbb{R}^3$.

(a)

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} \right\}$$

(b)

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} \right\}$$

(b)

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \right\}$$

2. (5 points) Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(\vec{x}) = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix} \vec{x}.$$

(a) Find a basis for $\operatorname{im} T$.

(b) Find a basis for $\ker T$.

(c) Find a linear transformation $F : V \to \mathbb{R}^3$ whose image is $\ker T$, and where F is one-to-one. You get to choose the vector space V.

3. (6 points) $\mathbb{P}_2$ is the vector space of polynomials of degree at most two, with real coefficients.

(a) Let S be the set of all polynomials from $\mathbb{P}_2$ whose derivative at 0 is 0 (that is, $p'(0) = 0$). Show that S is a vector subspace of $\mathbb{P}_2$.

(b) What is the dimension of S?

(c) Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be defined by $T(p) = p(x-1) - p(x)$. (For instance, $T(x^2+1) = ((x-1)^2+1) - (x^2+1)$.) What are $\ker T$ and $\operatorname{im} T$? Describe them by finding a basis for each.

4. (5 points) Let A be an $n \times m$ matrix and B be an $m \times n$ matrix such that $BA = I_m$.

(a) What is the dimension of $\operatorname{Col} B$?

(b) What is the dimension of $\operatorname{Nul} A$?

(c) Which of the following cannot happen? $n > m$ or $m > n$? Explain why not.

For fun. (0 points) Let A be an $n \times n$ matrix such that $A^2 = A$. Which vectors are in both $\operatorname{Col} A$ and $\operatorname{Nul} A$?