Quiz I Syllabus

Kyle Miller

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For studying, there are at least two aspects:

1. Know the definitions of each of the words we use. It's good to know ways of thinking about the words, but you should always know the concrete definitions.

2. Know the consequences of various facts (the big consequences being called "theorems"). These give meaning to the definitions. (An example: what does it mean when the null space of a matrix is equal to the zero subspace? There are a number of consequences!)

Find some way of committing them to memory: flash cards, say things out loud, discuss with a friend, etc. A large part of math proficiency really is linguistic.

Of course, there's also mechanically practicing certain calculations. (Make sure you can read your own writing!) Also give some thought that solving problems is a writing exercise, and so you have to consider your audience (in this case, a graduate student who is looking for evidence of satisfactory understanding).

This quiz will not be open book, and there will be no calculators or written resources (such as cheat sheets). It's good for studying to make a cheat sheet, and it might be comforting to have one in your backpack even if you can't use it.

1 Section 4.1

- The definition of **vector space**: the set of vectors, addition, scalar multiplication, and the properties ("axioms") these must together have.
- Examples of vector spaces: $\mathbb{R}^n$, $\mathbb{P}_n$, $\mathbb{P}$ (for arbitrary-degree polynomials), the set of real-valued continuous functions $\mathbb{R} \to \mathbb{R}$, etc.
- The definition of **subspace** (the three properties).
- The **zero subspace**.
- **Linear combination**, and **span**. That span is a subspace.
- Nonexamples of subspaces (for instance $\{(x, y) \in \mathbb{R}^2 : x + y = 1\}$).
- Examples of subspaces (for instance $W_1 \cap W_2$ for W_1 and W_2 subspaces of a vector space V).

2 Section 4.2

- **Null space** of an $m \times n$ matrix. How this is a subspace of $\mathbb{R}^n$.
- Describing null space as a span of vectors (via parametrized vector form via reduced echelon form).
- **Column space** of an $m \times n$ matrix. How this is a subspace of $\mathbb{R}^m$.
- That every subspace of $\mathbb{R}^m$ is a column space of some matrix.
- The definition of a **linear transformation** (the two properties).
- The definition of the **kernel** of a linear transformation $T : V \to W$ (denoted $\ker T$). That this is a subspace of V.
- The definition of the **range** or **image** of a linear transformation $T : V \to W$ (denoted $\operatorname{im} T$). That this is a subspace of W.
- The connection between kernel and null space, image and column space. As in, for $T : \mathbb{R}^n \to R^m$, then $\ker T = \text{Nul}[T]$ and $\operatorname{im} T = \text{Col}[T]$.

3 Section 4.3

- An indexed set of vectors being **linearly independent**. What if the indexed set contains 0? What if the indexed set contains the same vector twice?
- The definition of **linearly dependent**. A **linear dependence relation**.
- That a **basis** of a vector space V is a linearly independent spanning set.
- The spanning set theorem. ("If a vector in a (finite) spanning set is dependent on the others, it may be removed to get another spanning set." and "Every (finite) spanning set contains a subset which is a basis.")
- Finding bases for the null space and column space of a matrix.

4 Section 4.4

We haven't covered this section is discussion yet, but there are still fair questions that can be asked about concepts from this section.

1. Given a basis $\{b_1, \ldots, b_n\}$ of V, and a vector $v \in V$, find the coordinates of v relative to the basis. That is, find $c_1, \ldots, c_n \in \mathbb{R}$ such that $c_1 b_1 + \cdots + c_n b_n = v$.

5 Section 4.5

1. Finding the **dimension** of a subspace. The dimension is the number of vectors in a basis for the subspace.

2. The relationship between the dimension of $\operatorname{Nul} A$ and $\operatorname{Col} A$. Since $\operatorname{Nul} A$ has a basis whose elements are in correspondence with the free columns of A, and $\operatorname{Col} A$ the pivot columns, then the total number of columns of A is equal to the sum of the dimension of $\operatorname{Nul} A$ and the dimension of $\operatorname{Col} A$.

6 Section 4.6

1. The relationship between the dimension of $\operatorname{Nul} A$ and whether A is one-to-one.

2. The relationship between the dimension of $\operatorname{Col} A$ and whether A is onto.