MATH 54. October 28, 2016. (20 minutes, 10 points) Name:

Quiz 9

1. (5 points) (a) Find a vector $\vec{v}_3 \in \mathbb{R}^3$ which is orthogonal to both $\vec{v}_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.
(b) What are all the vectors orthogonal to $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$? (Hint: $\dim V^\perp$ for $V = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.)

2. (5 points) For $\vec{w} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $V = \text{Span}\left\{\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right\}$, find two vectors $\vec{w}^{\parallel} \in V$ and $\vec{w}^{\perp} \in V^\perp$ so that $\vec{w} = \vec{w}^{\parallel} + \vec{w}^{\perp}$. (In other words, decompose $\vec{w}$ into the projection/parallel component and the orthogonal component. The vector $\vec{w}^{\parallel}$ is $\text{proj}_V \vec{w}$.)

(For fun) (a) Why is every square matrix a “root” of a polynomial? (For instance, $A = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 1 \end{smallmatrix}\right)$ satisfies $I + A + A^2 = 0$, so it is the “root” of $1 + x + x^2$.) (b) Let $p(x)$ be a polynomial where $p(A) = 0$. Why is every eigenvalue λ of A a root of p? (Hint: use an eigenvector $\vec{v}$ to show $p(A)\vec{v} = p(\lambda)\vec{v}$.) (c) If P is the matrix of proj_V, then $P^2 = P$ since projecting a projected vector does not change the vector. This means $P^2 - P = 0$. What are the possible eigenvalues of P?