Quiz 7

1. (5 points) Suppose $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ are two bases of some vector space V, with $\vec{b}_1 = 3\vec{c}_1 + 5\vec{c}_2$ and $\vec{b}_2 = \vec{c}_1 + 2\vec{c}_2$.

   (a) Find the change-of-coordinates matrix P from $\mathcal{B}$ to $\mathcal{C}$. (In other words, a P which satisfies $P[\vec{x}]_{\mathcal{B}} = [\vec{x}]_{\mathcal{C}}$ for all $\vec{x} \in V$.)

   (b) Find the change-of-coordinates matrix Q from $\mathcal{C}$ to $\mathcal{B}$.

   (c) What is QP equal to?

2. (5 points) Suppose $\vec{v} \in \mathbb{R}^4$ and $\vec{w} \in \mathbb{R}^3$, both nonzero, and let $A = \vec{v}\vec{w}^T$.

   (a) What is the size of A? (Give as $n \times m$ for some n and m.)

   (b) What is rank A?

   (c) What is dim Nul A?

(For fun) What is the dimension of the subspace of polynomials from $\mathbb{P}_2$ which have 1 as a root?