Quiz 6

1. Let $W = \{A \in \mathbb{R}^{2\times 2} : A^T = A\}$ (the set of 2×2 symmetric matrices).

   (a) (4 points) Show that W is a subspace of $\mathbb{R}^{2\times 2}$.

   (b) (1 point) Find a basis for W.

2. (5 points) For the following set W, (a) find a matrix A so that $W = \operatorname{Col} A$, (b) explain why W is a subspace, and (c) give its dimension.

$$W = \left\{ \begin{pmatrix} a + 2b + 4c \\ b + 3c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$$

(For fun) A square matrix A which satisfies $A^T = A$ is called *symmetric*, and a square matrix A which satisfies $A^T = -A$ is called *antisymmetric*. Give a way to write any given square matrix as a sum of a symmetric and an antisymmetric matrix. Is this representation unique?