

Quiz 5

1. (5 points) Suppose A is a square matrix.

- (a) If $\det(A^4) = 0$, explain why A cannot be invertible.
- (b) If A is invertible, explain why $\det(A^T A) > 0$.

2. (5 points) Let $A = \begin{pmatrix} 1 & 1 & 3 \\ 2 & -2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

- (a) Compute $\det A$.
- (b) Compute the entry in row 2 column 3 of A^{-1} using the adjugate of A (or Cramer's rule).

(For fun) What is the determinant of the following matrix (called a *Vandermonde* matrix)? You should be able to factorize it into a product of three very simple terms.

$$\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$$

Using the determinant, what are the conditions on a, b, c for the matrix to be invertible?