Quiz 4

1. (3 points) For each statement following each matrix shape, circle YES if there is a matrix with that shape where the statement is true, and NO if there is no such matrix.

(a) For a 3×3 matrix:

(i) The columns might span $\mathbb{R}^3$: YES / NO
(ii) The columns might not span $\mathbb{R}^3$: YES / NO
(iii) The columns might be independent: YES / NO
(iv) The columns might not be independent: YES / NO

(b) For a 3×4 matrix:

(i) The columns might span $\mathbb{R}^3$: YES / NO
(ii) The columns might not span $\mathbb{R}^3$: YES / NO
(iii) The columns might be independent: YES / NO
(iv) The columns might not be independent: YES / NO

(c) For a 3×2 matrix:

(i) The columns might span $\mathbb{R}^3$: YES / NO
(ii) The columns might not span $\mathbb{R}^3$: YES / NO
(iii) The columns might be independent: YES / NO
(iv) The columns might not be independent: YES / NO

2. (5 points) For each of the following matrix shapes, complete the corresponding table with all possibilities for $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$. Cross out impossible lines.

(a) A is 4×4

$\operatorname{rank} A$	$\dim \operatorname{Nul} A$
0	
1	
2	
3	
4	

(b) A is 2×4

$\operatorname{rank} A$	$\dim \operatorname{Nul} A$
0	
1	
2	
3	
4	

(c) A is 4×2

$\operatorname{rank} A$	$\dim \operatorname{Nul} A$
0	
1	
2	
3	
4	

3. (2 points) Compute $\det \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 3 & 4 & 0 \end{pmatrix}$.

(For fun) Is the inverse of an invertible upper triangular matrix always upper triangular?