Quiz 3

1. (5 points) The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_2, x_1 + x_3, x_2 - x_1)$.
   (a) Write T as a matrix transformation.
   (b) Is T injective (one-to-one)?
   (c) Is T surjective (onto)?

2. (5 points)
   (a) Compute the inverse A^{-1} for $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 8 \end{pmatrix}$.
   (b) Solve $AX = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & 5 \end{pmatrix}$ for the 3×3 matrix X.

(For fun) For an $n \times m$ matrix A and an $m \times n$ matrix B, show that if the columns of A do not span $\mathbb{R}^n$, then neither do the columns of AB. (But, if the the columns of B span $\mathbb{R}^m$, then the span of the columns of A equals the span of the columns of AB.)

(For fun) If A is $n \times n$ with $A^T A = I_n$, how can you solve $A\vec{x} = \vec{b}$ without computing A^{-1}?