MATH 54. December 2, 2016. (20 minutes, 10 points) Name: ____

Quiz 13

1. (5 points) Let $A = \begin{pmatrix} 8 & -9 \\ 6 & -7 \end{pmatrix}$. (a) Give the general solution for $\vec{x}'(t) = A\vec{x}(t)$. (b) Solve the initial value problem for $\vec{x}(0) = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$.

2. (5 points) The heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t) = u(1,t) = 0$ and $u(x,0) = f(x)$ for $0 < x < 1$ has the solution $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-(n\pi)^2 t} \sin(n\pi x)$ for c_n's satisfying $f(x) = \sum_{n=1}^{\infty} c_n \sin(n\pi x)$. Give the solution for when $f(x) = 2\sin(\pi x) - 5\sin(3\pi x) + 8\sin(7\pi x)$.

(For fun) Old console games frequently used *square waves* for their soundtracks. For instance, let $f(x)$ be a periodic function with period 2π which we define on $[-\pi, \pi]$: let $f(x) = -1$ if $-\pi < x < 0$ and $f(x) = 1$ if $0 < x < \pi$. Use the integral inner product $\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)\,dx$ to write $f(x)$ as a linear combination $f(x) = \sum_{n=1}^{\infty} c_n \sin(nx)$. (This is a *Fourier series*. For sound, each term is called a *harmonic*.)