Midterm 2 schemata - These are some of the ingredients for questions on a midterm. (Elements from Midterm 1 still apply.)

- Determine whether a matrix is invertible, in whatever form that might take (compile a complete Invertible Matrix Theorem).
- Use determinant rules, for instance to show $\det(PBP^{-1}) = \det(B)$.
- Use Cramer's rule to find a single entry x_i of a solution to $A\vec{x} = \vec{b}$, or a single entry of A^{-1}.
- Given a set, tell whether or not it is a subspace (check the three properties). Know examples of vector spaces: $\mathbb{R}^n$, $\mathbb{P}_n$, $\mathbb{R}^{m\times n}$, $\{f : \mathbb{R} \to \mathbb{R} : f \text{ continuous}\}$.
- Recognize a set as $\operatorname{Nul} A$, $\operatorname{Col} A$, $\ker T$, or $\operatorname{im} T$; use this to conclude the set is a subspace (to skip needing to check the three properties). Finding bases of $\operatorname{Nul} A$, $\operatorname{Col} A$, $\operatorname{Row} A$, $\ker T$, or $\operatorname{im} T$.
- Show whether a transformation $T : V \to W$ is a *linear* transformation. (Check the two properties.)
- Relationship between injective/one-to-one and $\operatorname{Nul} A$ or $\ker T$. Relationship between surjective/onto and $\operatorname{Col} A$, $\operatorname{im} T$. Relationship between dimensions of $\operatorname{Nul} A$ and $\operatorname{Col} A$, or of $\ker T$ and $\operatorname{im} T$.
- Check whether a set is a basis. Find the dimension of a subspace. That the dimension of a subspace W of V cannot exceed the dimension of V.
- Take a spanning set and make a basis (spanning set theorem). Take an independent set and make a basis (add vectors from, say, the standard basis to the end to make it a spanning set, then use the spanning set theorem).
- Given a vector of V and a basis $\mathcal{B}$, find the coordinate relative to $\mathcal{B}$. Given a coordinate vector, find the corresponding vector of V.
- Given two bases $\mathcal{B}$ and $\mathcal{C}$, find the basis change matrix from $\mathcal{B}$-coordinates to $\mathcal{C}$-coordinates.
- That the coordinate mapping $\vec{x} \mapsto [\vec{x}]_{\mathcal{B}}$ is an isomorphism.
- Given a matrix A in $\mathcal{B}$-coordinates of a transformation T and a basis change matrix P from $\mathcal{B}$-coordinates to $\mathcal{C}$-coordinates, that PAP^{-1} is the matrix of T in $\mathcal{C}$-coordinates.
- The matrix of a transformation relative to a pair of bases for the domain and codomain.
- Check whether a vector is an eigenvector, and recover its eigenvalue. That $\vec{0}$ is never ever an eigenvector.
- Calculate the eigenvalues of a matrix using the characteristic polynomial.
- That $A - \lambda I$ is not invertible exactly when λ is an eigenvalue.
- Determining whether a matrix is diagonalizable, either by finding a basis of eigenvalues or noticing the matrix has distinct eigenvalues, whichever works. Diagonalizing a matrix (that is, giving P and D for $A = PDP^{-1}$). Understanding that the columns of P are a basis of eigenvectors.
- Using the correspondence between A being invertible and A not having 0 as an eigenvalue.
- Finding the eigenvalues of A^{-1}, A^T, or $(A^T)^{-1}$ from the eigenvalues of A.
- Computing A^n from a diagonalization of A. Or computing $A^n\vec{v}$ from a diagonalization of A. Or, simpler, when $\vec{v}$ is a known eigenvector of A.

- Finding the eigenvalues of a linear transformation $T : V \to V$ by first taking any basis of V, and then finding the eigenvalues of the matrix of T relative to the basis.
- Using the dot product properties (Theorem 1, 6.1). Length, normal vectors. Pythagorean theorem or, possibly, the triangle inequality.
- Compute the basis of the orthogonal complement of a subspace. $(\operatorname{Col} A)^{\perp} = \operatorname{Nul} A^T$.
- Orthogonal sets of nonzero vectors are linearly independent. Orthogonal basis. Orthonormal basis. Finding the coordinates of a vector relative to an orthonormal/orthogonal basis.
- Use the fact U^T is the inverse matrix of a square orthogonal matrix U. Use the fact that for any orthogonal matrix U, $(U\vec{x}) \cdot (U\vec{y}) = \vec{x} \cdot \vec{y}$.
- Computing the projection of a vector onto a subspace.