Midterm 1 schemata - These are some of the ingredients for questions on a midterm.

- Given a matrix A, compute bases/dimensions for $\operatorname{Col} A$, $\operatorname{Nul} A$, maybe $\operatorname{Row} A$.
- Given a set of vectors, determine whether they are independent or whether they span $\mathbb{R}^n$.
- Given a subspace W of $\mathbb{R}^n$, compute basis/dimension.
- Given spanning set for subspace, compute basis/dimension.
- Give solution set to $A\vec{x} = \vec{b}$ in parametric vector form.
- Determine whether a vector is a linear combination of a set of vectors; similarly, determine whether $A\vec{x} = \vec{b}$ has a solution. Or, tell whether $A\vec{x} = \vec{b}$ has at most one solution.
- Give solution set to $A\vec{x} = \vec{0}$ as a span.
- Given independent set of vectors, find a vector not in the span of the vectors (if one exists).
- Given a transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, determine whether it is a linear transformation. (If it is, show the two properties, if it isn't, give a counterexample where the properties are violated.)
- Use linearity properties of a linear transformation (example: show $T(\vec{0}) = \vec{0}$ for any linear transformation T).
- Given a subset W of $\mathbb{R}^n$, determine whether it is a subpsace. If it is, demonstrate the three properties, and if it isn't, given a concrete counterexample where a property is violated.
- Compute matrix of a transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.
- Compute coordinate vector (a.k.a. weights) of a vector relative to a given basis, or given coordinate vector, find the corresponding vector. In other words, if the columns of A are a basis, use $A\vec{c} = \vec{x}$ to transform between a coordinate $\vec{c}$ and a vector $\vec{x}$.
- Determine whether a square matrix is invertible. Compute the inverse of a matrix.
- Determine whether a matrix A has $\vec{x} \mapsto A\vec{x}$ onto (surjective), one-to-one (injective), or both (bijective). Know how this is related to pivots, spanning, and independence.
- Given a matrix A, compute $\operatorname{rank} A$ or $\dim \operatorname{Nul} A$, or compute one from the other using the rank theorem.
- Given an $m \times n$ matrix A, determine whether there is an $n \times m$ matrix B or $n \times m$ C so that $BA = I_n$ or $AC = I_m$. Compute such a matrix.
- Compute basis/dimension of $\ker T$ or $\operatorname{im} T$ given some linear transformation T. (This is if the kernel of T has appeared. If A is the matrix of T, $\operatorname{im} T = \operatorname{Col} A$ and $\ker T = \operatorname{Nul} A$.)
- Compute rank/invertibility/spanning columns/independent columns of AB, A, or B, given information about AB, A, and/or B.
- Use the row echelon form algorithm to deduce something about pivots of a matrix. (For instance, $\operatorname{rank} A \leq \operatorname{rank} [A \mid \vec{b}] \leq 1 + \operatorname{rank} A$ could be solved this way, though is probably trickier than what you would see on the midterm.)
- Write row operations as elementary matrices. How the result of row operations can be written as $R = E_k \dots E_2 E_1 A$ with $E_1, \dots, E_k$ the elementary matrices and R a matrix in (reduced) row echelon form. How elementary matrices are invertible. How this decomposition lets you write any invertible matrix as a product of elementary matrices.