Discussion – Nov 21

Understanding the Wronskian

1. a) Let $f_1(t)=c_1e^{\lambda_1 t}$ and $f_2(t)=c_2e^{\lambda_2 t}$. Compute $W[f_1,f_2](t)$.
   b) Under what condition is there a t_0 where $W[f_1,f_2](t_0)=0$?
   c) If $W[f_1,f_2](t)=0$ for some t, what function is $W[f_1,f_2](t)$?
   d) Find a 2^{nd} order linear homog. diff. eq. both f_1 and f_2 are a solution to.
2. a) Let $f_1(t)=c_1e^{\lambda t}$ and $f_2(t)=c_2t^2e^{\lambda t}$. Compute $W[f_1,f_2](t)$.
   b) Is there a t_0 where $W[f_1,f_2](t_0)=0$?
   c) Yet is $W[f_1,f_2](t)$ the zero function? Are f_1,f_2 independent?
   d) Are f_1,f_2 solutions to the same 2^{nd} order linear homog. diff. eq.?
   e) Find a 3^{rd} order linear homog. diff. eq. having f_1,f_2 as solns.
3. a) Let $f_1(t)=t^2$ and $f_2(t)=t|t|$. Compute $W[f_1,f_2](t)$.
   b) Is there a t_0 where $W[f_1,f_2](t_0)=0$?
   c) Is $W[f_1,f_2](t)$ the zero function?
   d) Are f_1,f_2 independent? (Hint: evaluate at some points.)
   e) Yet are f_1,f_2 solutions to the same homog. diff. eq.?
   (Hint: find a point where they have the same init. conditions)
4. For $f_1,\dots,f_n$ some functions, put the following pieces together:
   i) $f_1,\dots,f_n$ are solns to same n^{th} order homog. lin. diff. eq. w/ const coeff.
   ii) $f_1,\dots,f_n$ are linearly independent.
   iii) There is a t_0 such that $W[f_1,\dots,f_n](t_0)=0$.
   iv) For all t, $W[f_1,\dots,f_n](t)=0$.
   (Make a Venn diagram! Have an example for each region!)

5. Solve $y'''' - y''' - 3y'' + 5y' - 2y = 0$
6. Solve $y'' + y = \sec t$