Discussion — Nov. 9

1. Consider $\mathbb{P}_2$ with the inner product $\langle f,g\rangle = \int_0^1 f(x)g(x)\,dx$.
   a) Perform Gram-Schmidt on $\{1, x, x^2\}$ to get an orthogonal basis $\mathcal{B}$.
   b) Compute $[1-x^2]_{\mathcal{B}}$.
2. Consider $\mathbb{P}_2$ with the inner product $\langle f,g\rangle = \sum_{k=0}^{2} f(k)g(k)$.
   a) Do (a) again with this.
   b) Use this basis to find a polynomial $p(x)$ such that $p(0)=0$, $p(1)=1$, $p(2)=3$
3. Consider $\mathbb{R}^3$ with dot product. Find collections of unit vectors where the minimum distance between any two is maximized. (a) two vectors (b) 3 (c) 4 (d) 6 (e) 5
4. Solve $x'' - 3x' + 2 = 0$. a) generally b) $x(0)=1$ $x'(0)=0$
5. Solve $x'' - 2x' + 1 = 0$. a) generally b) $x(0)=1$ $x'(0)=0$