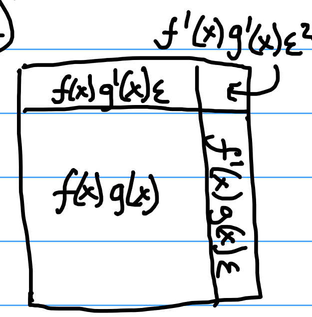
# Discussion — Nov 7

## Calculus review

1. Here's a nonstandard way to do calculus. Let $\varepsilon$ represent an infinitesimal, a "fake" number with the properties $\varepsilon^2 = 0$ and if $c \in \mathbb{R}$ with $c > 0$, $0 < \varepsilon < c$ (it is smaller than all positive numbers). Derivative rule: $f(a + b\varepsilon) = f(a) + bf'(a)\varepsilon$.
   (a) Check $(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)$
   (b) Check $(1/f(x))' = -f'(x)/f(x)^2$
   (c) Find a rule for $(f(x)g(x)h(x))'$
   (d) Check $(f(g(x)))' = f'(g(x))\,g'(x)$
   (e) Check $(cf(x))' = cf'(x)$
   (f) Check $(f(x)+g(x))' = f'(x) + g'(x)$




2. Complete the tables

| $f(x)$ | $f'(x)$ |
|---|---|
| $c$ | |
| $x$ | |
| $x^n$ | |
| $\sqrt{x}$ | |
| $\sin(cx)$ | |
| $\cos(cx)$ | |
| $e^{cx}$ | |

| | $f'(x)$ | $f(x) + c$ |
|---|---|---|
| | $c$ | |
| | $x$ | |
| $(n \neq -1)$ | $x^n$ | |
| | $x^{-1}$ | |
| | $\sqrt{x}$ | |
| | $\sin(cx)$ | |
| | $\cos(cx)$ | |
| | $e^{cx}$ | |

3. Solve the differential equation $f' = \lambda f$ (also written as $\frac{d}{dx}f = \lambda f$ or $\frac{df}{dx} - \lambda f = 0$)
4. Draw vector fields for $f' = f$ and $f' = -f$.
5. Try drawing the graph of a solution to $f'' - f' - 6f = 0$ with $f(0) = 1$ and $f'(0) = 0$ by contemplating concavity.
6. Solve the differential equation $f'' - f' - 6f = 0$ (take a leap by factoring out $\frac{d}{dx}$!)