Discussion - Nov 2

1. Let $\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, and let $V = \text{Span}\{\vec{u}_1, \vec{u}_2\}$.
(a) Find the standard matrix of $T(\vec{x}) = \text{proj}_V \vec{x}$.
(b) What is the nullspace of the matrix?
(c) What <u>is</u> the nullspace of the matrix?
(d) Why is the matrix symmetric? $(A = A^T)$

2. Consider the data in table 1. (a) Is there a line which passes through all the points? Make a system of three equations $y = mx + b$ with (x, y) from the table, m, b the unknowns. Show it is inconsistent.

x	y
0	1
1	1
1	2

table 1

(b) Writing the system as $A\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \end{bmatrix}$, find the "least squares" solution. Either do it from scratch, or recall you solve $A^TA\begin{bmatrix} m \\ b \end{bmatrix} = A^T\begin{bmatrix} y_1 \\ \vdots \end{bmatrix}$. (c) Plot the resulting $y = mx + b$.

3. Let's find an orthonormal basis of $\mathbb{R}^3$ involving $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.
(a) Extend $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ into a basis of $\mathbb{R}^3$: take the pivot columns of $\begin{bmatrix} \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} & \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \end{bmatrix}$. (b) Perform Gram-Schmidt to get an orthogonal basis from this. (c) Normalize the vectors. (d) Why is step (a) optional? Could we do Gram-Schmidt on $\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{e}_1, \vec{e}_2, \vec{e}_3\}$ instead? (Try it.) (e) Check the determinant of your matrix — if it is negative, swap the second two vectors. Graph the vectors to make sure it has the right hand rule.
(f) Rotation CCW around $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ by θ in this basis has matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$. What is the matrix of the rotation in the standard basis? (Hint: $R = UAU^T$).

4. $A = QR$ with Q having orthonormal columns and R square upper triangular with positive diagonal entries is a <u>QR factorization</u>.
(a) Why is $\text{Col } A = \text{Col } Q$? (b) Suppose you do Gram-Schmidt on the columns of A to get an orthonormal set Q. Why is R then Q^TA? (c) How does QR help solve $A\vec{x} = \vec{b}$? (d) least squares?