Discussion – Oct 26

1. Let $\mathcal{S} = \{(c_0, c_1, c_2, \ldots) \mid c_i \in \mathbb{R}\}$, the vector space of sequences.
   (a) Let $\sigma : \mathcal{S} \to \mathcal{S}$ be $\sigma(c_0, c_1, \cdots) = (c_1, c_2, \ldots)$, the shift operator. Check it is a linear transformation.
   (b) Find a sequence c where $\sigma(c_0, \cdots) = 2(c_0, \ldots)$.
   (c) What are all the eigenvalues of σ? eigenvectors?
   (d) Let $F = \sigma^2 - \sigma - \mathbb{1}$. Show the fibonacci sequence $(1, 1, 2, 3, 5, \cdots)$ is in $\ker F$.
   (e*) Factorize F as $(\sigma - \lambda_1)(\sigma - \lambda_2)$.
   (f*) Write the fibonacci sequence as a lin. comb. of e.vecs associated with λ_1 and λ_2.
2. What are the eigenvectors of $\frac{d}{dx}$?
3. Find the set of vectors orthogonal to $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\2\\3\end{pmatrix}$
4. Find the set of vectors orthogonal to $\begin{pmatrix}1\\2\\3\end{pmatrix}$
5. Normalize $\begin{pmatrix}3\\4\end{pmatrix}$
6. Find the distance between $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$
7. If A has linearly indep. columns, is A^TA invertible?
8. For $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & -2 \end{pmatrix}$, find the matrix of
   $$T : \mathbb{R}^3 \to \mathbb{R}^3 \text{ def. by } T(\vec{x}) = \begin{bmatrix} \vec{a}_1 \cdot \vec{x} \\ \vec{a}_2 \cdot \vec{x} \\ \vec{a}_3 \cdot \vec{x} \end{bmatrix}$$
9. Why is $(\mathrm{Col}\, A)^\perp = \mathrm{Nul}\, A^T$?
10. If U orthogonal matrix, why is $(U\vec{x}) \cdot (U\vec{y}) = \vec{x} \cdot \vec{y}$?