Discussion – Oct. 24

1. $\mathbb{C} = \mathrm{Span}\{1, i\}$ (where $i^2 = -1$) is the number system called the <u>complex numbers</u> (com = together, plex = tied), extending $\mathbb{R}$ with an <u>imaginary unit</u> i.
   (a) Convince yourself $\mathbb{R}$ is a subspace of $\mathbb{C}$, and $\dim \mathbb{C} = 2$.
   (b) Check $T(z) = iz$ is a linear transformation. $(T: \mathbb{C} \to \mathbb{C})$
   (c) Find the matrix of T relative to basis $\{1, i\}$.
   (d) What kind of matrix is it? (Geometrically speaking)
   (e) Find the eigenvalues. Diagonalize it as PDP^{-1} with P and D matrices with complex entries.

2. Diagonalize $A = \begin{pmatrix} -10 & -18 \\ 6 & 11 \end{pmatrix}$.

3. Why can you check a diagonalization by seeing whether $AP = PD$? Why is this easier than the alternative?

4. For $A = PDP^{-1}$ with $P = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, give at least two different diagonalizations of A.

5. Diagonalize
$$A = \begin{pmatrix} 5/3 & 0 & 2/3 \\ 0 & 3 & 0 \\ 4/3 & 0 & 7/3 \end{pmatrix}$$

6. Which of the following are diagonalizable?
   (a) $A = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 2 \end{pmatrix}$ (b) $A = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (c) $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 1 & 1 & 0 \\ & 2 & 1 \\ & & 3 \end{pmatrix}$ (e) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

7. Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$. Compute A^{10}.

   (Can do without diagonalizing by $A^{2^3} A^{2^1}$, $A^{2^3} = ((A^2)^2)^2$.)

8. Suppose A is $n \times n$ with char. poly λ^n. Is $I - A$ invertible?