Discussion – Oct. 19

1. For each of the following, determine whether the statement is true always, sometimes, or never. ("sometimes" requires an example where true and an example where false.)
   (a) Square matrices a/s/n are invertible.
   (b) Square matrices a/s/n are diagonalizable.
   (c) Diagonal matrices a/s/n are diagonalizable.
   (d) Matrices A with $\det A = 0$ a/s/n are diagonalizable.
   (e) Diagonalizable matrices a/s/n are invertible.
   (f) 0 is a/s/n an eigenvalue.
   (g) For a non-invertible matrix, 0 is a/s/n an eigenvalue.
   (h) The $\vec{0}$ vector is a/s/n an eigenvector.
   (i) If $\vec{v}_1, \ldots, \vec{v}_k$ are eigenvectors corresponding to distinct eigenvalues, then they are a/s/n independent.
   (j) If $\vec{v}_1, \ldots, \vec{v}_k$ are linearly independent, they are a/s/n eigenvectors.
   (k) If $\vec{v}_1, \ldots, \vec{v}_k$ are linearly indep. eigenvectors, they a/s/n correspond to distinct eigenvalues.
   (l) A and A^T a/s/n have the same eigenvalues with the same multiplicity.
   (m) if A is $n \times n$ it a/s/n has all real eigenvalues.
   (n) if A is $n \times n$ with n odd, it a/s/n has at least one real eigenvalue.

2. If A is diagonalizable, how can you compute A^n easier?

3. Find a 2×2 matrix A with $A\begin{pmatrix}1\\2\end{pmatrix} = 3\begin{pmatrix}1\\2\end{pmatrix}$ and $A\begin{pmatrix}2\\-1\end{pmatrix} = -\begin{pmatrix}2\\-1\end{pmatrix}$. Hint: A is diagonalizable.