

Discussion - July 11

1. Check that $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\begin{pmatrix} 5 & -3 \\ 1 & 1 \end{pmatrix}$.
What is its eigenvalue?
2. What are eigenvalues/eigenvectors of $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which is reflection over $y=x$?
3. Does rotation of \mathbb{R}^2 by 90° CCW have any (real) eigenvalues?
4. An early discussion problem was for which λ does
$$\begin{cases} (\lambda-3)x + y = 0 \\ x + (\lambda-3)y = 0 \end{cases}$$
 have non-unique solutions.
What are eigenvalues and eigenvectors of $\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$?
5. Do all 3×3 matrices have 3 distinct eigenvalues?
6. If $\vec{0}$ were an eigenvector, what would its eigenvalue be? (This is supposed to show why we say "nonzero!")
7. What is the maximum number of eigenvalues an $n \times n$ matrix may have?
8. $A = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & 2 \end{bmatrix}$. Find eigenvalues and bases for eigenspaces (i.e., eigenvectors)
9. Find a 2×2 matrix with one 2-dim. eigenspace.
10. A is 4×4 with eigenvalues 1, 2, 3, 4. What is $\det(A)$?
11. $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ which reflects through xy plane.
Eigenvalues and eigenspaces?
12. $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ def. by $\vec{x} \mapsto \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$. Eigenvalues and eigenspaces?