Discussion – Oct. 5

1. Suppose $T: V \to W$ is a linear transformation, and $\{\vec{v}_1, \dots, \vec{v}_n\}$ is a set of vectors from V. Show if $\{T(\vec{v}_1), \dots, T(\vec{v})\}$ are independent, so are $\{\vec{v}_1, \dots, \vec{v}_n\}$.

2. Suppose V is some subspace of real-valued functions, and $T_\alpha: V \to \mathbb{R}$ is evaluation at some α $(T_\alpha(f(x)) = f(\alpha))$.

(i) Show T_α is a linear transformation.

(ii) Let $\alpha_1, \dots, \alpha_n$ be n different points. Show

$$T(f(x)) = \begin{bmatrix} f(\alpha_1) \\ \vdots \\ f(\alpha_n) \end{bmatrix} \qquad (V \to \mathbb{R}^n) \text{ is a linear transformation}$$

(iii) Show $\{\cos x, \sin x\}$ is an independent set.

(iv) Show $\{1, x, x^2\}$ is an independent set.

3. Use $\vec{e}_1, \vec{e}_2, \vec{e}_3$ to find a matrix A (3×3) so that in

$$\begin{tikzcd} \mathbb{R}^3 \arrow[r, "A"] \arrow[d, "T"'] & \mathbb{R}^3 \arrow[d, "T"] \\ \mathbb{P}_2 \arrow[r, "\frac{d}{dx}"] & \mathbb{P}_2 \end{tikzcd} \qquad \text{with } T(\vec{c}) = c_1 + c_2 x + c_3 x^2$$

this is just a graphical representation of

$$\frac{d}{dx}(T(\vec{c})) = T(A\vec{c}) \quad \text{for all } \vec{c}.$$

(A is the matrix of $\frac{d}{dx}$ relative to basis $\{1, x, x^2\}$)

4. One basis for Col A is the pivot columns of A. Why can you also take the nonzero rows of ref A^T?

5. Let $T: \mathbb{R}^{2\times2} \to \mathbb{R}^{2\times2}$ be $T(A) = \frac{1}{2}(A + A^T)$.

(i) What is ker T? basis?

(ii) What is im T? basis?

(iii) What is $T(T(A))$? What is $A - T(A)$?

(iv) Is T a linear transformation?