# Discussion — Oct. 3

1. For each of the following sets, determine whether it (i) has a zero vector (ii) is closed under addition (iii) is closed under scalar mult.

(a) {odd integers} (b) {even integers} (c) $\{A \in \mathbb{R}^{2\times 2} \mid \det A = 1\}$
(d) $\{A \in \mathbb{R}^{2\times 2} \mid a_{21} = 0\}$ (e) $\{A \in \mathbb{R}^{3\times 3} \mid$ entries of $A$ all negative$\}$
(f) For some $B \in \mathbb{R}^{3\times 2}$, $\{A \in \mathbb{R}^{2\times 2} \mid BA = 0\}$ (g) $\{A \in \mathbb{R}^{2\times 2} \mid A^4 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\}$
(h) $\{p(x) \in \mathbb{P}_2 \mid p(3) = 0\}$ (i) $\{p(x) \in \mathbb{P}_2 \mid p(3) = 1\}$
(j) $\{f(x)$ continuous $\mid f(3) = 0\}$ (k) $\{f(x)$ continuous $\mid f(3) = 1\}$
(l) $\{f(x)$ differentiable $\mid f'(3) = 0\}$ (m) $\{f(x)$ differentiable $\mid f'(x) = 0\}$
(n) $\{f(x) \mid f(x) = f(-x)\}$ (o) $\{f(x) \mid f(x) = -f(-x)\}$
(p) $\{f(x) \mid$ Taylor series at $x=0$ has radius of convergence $> 0\}$

2. Find an $A$ so that the set is (i) Nul $A$ (ii) Col $A$

(a) $\left\{ \begin{pmatrix} 3a \\ 2b \\ a+b \end{pmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$ (b) $\left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \;\middle|\; a + 2b = c \right\}$

(c) $\left\{ \begin{pmatrix} a+2b \\ c \\ a+c \end{pmatrix} \;\middle|\; \begin{matrix} a+b+c = 0 \\ 2a - c = 0 \end{matrix} \right\}$

3. For $A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$, come up with a $B$ so that Col $B$ = Nul $A$. Without multiplying, how do you know that $AB = 0$? (Check it.)

4. For $A$ $2\times 2$, show $\{I, A, A^2\}$ is always a dependent set.