Discussion – Sep 28

1. Find a pair of matrices where $\det(A+B) \neq \det(A) + \det(B)$.
2. Find a pair of matrices A and B where $\det(A)=0$, $\det(B)=0$, and $\det(A+B)=1$.
3. For A $n \times n$, what is the relationship between $\det(cA)$ and $\det(A)$?
4. Compute $\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Is $\dim \mathrm{Nul}(A^T) = 0$?
5. Solve $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ using Cramer's rule.
6. Compute $\begin{pmatrix} x & -1 \\ 1 & x \end{pmatrix}^{-1}$ using $\left(\frac{1}{\det A} \operatorname{adj} A\right) A = I$ (consq. of Cramer's rule)
   For which x is there an inverse. (And is $x \mapsto A(x)$ continuous?)
7. Using determinants, how can you tell if three vectors in $\mathbb{R}^3$ lie on the same a) plane b) line? (Geom. interp might help)
8. Compute determinants and decide whether the matrix is invertible.
   a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix}$ b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$

   d) $\begin{pmatrix} 2 & 3 & 0 \\ 1 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 4 & 1 \\ 0 & 1 & 2 & 2 \\ 3 & 0 & -1 & 7 \end{pmatrix}$ f) $\begin{pmatrix} a & * & * \\ 0 & b & * \\ 0 & 0 & c \end{pmatrix}$
9. Solve for x_2 using Cramer's rule
   $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 \\ 1 & 4 & 9 & 2 \end{array}\right)$ (check it with row reduction)
10. Determinants are positive or negative.
    a) Find some examples of pairs of vectors in $\mathbb{R}^2$ whose det is ± 1. Draw them.
    b) do the same for $\mathbb{R}^3$
    c) Why might people call the sign of det the "orientation"?