Discussion – Sep 19

Definitions you should know:

- $\mathrm{Span}\{\vec{a}_1, \cdots, \vec{a}_n\} = \{c_1\vec{a}_1 + \cdots + c_n\vec{a}_n \mid c_1, \ldots, c_n \in \mathbb{R}\}$
  <u>or</u> $= \{A\vec{c} \mid \vec{c} \in \mathbb{R}^n\}$ with $A = [\vec{a}_1 \cdots \vec{a}_n]$

  fact: this is a subspace of $\mathbb{R}^m$ (assuming $\vec{a}_1, \ldots, \vec{a}_n \in \mathbb{R}^m$)
- A set of vectors is a <u>spanning set</u> if ________.
- A set of vectors is an <u>independent set</u> if $\underline{\quad \cdots \text{(non-trivial)} \cdots \quad}$.
- $\mathrm{Nul}\, A = \{\vec{x} \in \mathbb{R}^n \mid A\vec{x} = \vec{0}\}$ with A $m \times n$
- $\mathrm{Col}\, A = \{A\vec{x} \mid \vec{x} \in \mathbb{R}^n\}$ with A $m \times n$
- A <u>subspace</u> W of $\mathbb{R}^n$ is a subset of $\mathbb{R}^n$ ($W \subset \mathbb{R}^n$) such that a) $\vec{0} \in W$ b) $\vec{a}, \vec{b} \in W \Rightarrow \vec{a} + \vec{b} \in W$ c) $\vec{a} \in W, c \in \mathbb{R} \Rightarrow c\vec{a} \in W$.
- A <u>basis</u> of a subspace W is a linearly independent spanning set of vectors from W ($\mathbb{R}^n$ has <u>standard basis</u> $\vec{e}_1, \ldots, \vec{e}_n$)
- The <u>dimension</u> of a subspace is the number of vectors in a basis.
- $\mathrm{rank}\, A = \dim \mathrm{Col}\, A$

  fact: $\mathrm{rank}\, A + \dim \mathrm{Nul}\, A = n$ (A is $m \times n$)

  \# pivot cols + \# free cols = \# cols

fact: the columns of an $n \times n$ invertible matrix are a basis of $\mathbb{R}^n$ (why?)

fact: given a basis $\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_p\}$ for a subspace $W \subset \mathbb{R}^n$, every vector $\vec{x} \in W$ can be written as $\vec{x} = c_1\vec{b}_1 + \cdots + c_p\vec{b}_p$ for some $\vec{c} \in \mathbb{R}^p$ in exactly one way (why?)

notation: $[\vec{x}]_{\mathcal{B}} = \vec{c}$.

fact: both $\mathrm{Nul}\, A$ and $\mathrm{Col}\, A$ have a finite basis.
(How do you calculate them?)

fact: if $\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_p\}$ is a basis of subspace $W \subset \mathbb{R}^n$, then $W = \mathrm{Span}\{\vec{b}_1, \ldots, \vec{b}_p\}$ (why?)