Discussion – Sep 14

1. (i) When multiplying a 2×3 and a 3×4 matrix together, how many multiplications does it take? (real number mults.)
   (ii) What about $\ell\times m$ times $m\times n$?
   (iii) Suppose you have to calculate ABC with A 2×3, B 3×4, and C 4×5. Which takes less work to compute (fewer mults.)? $(AB)C$ or $A(BC)$?
2. Find a pair of 2×2 matrices A, B with $AB \neq BA$
3. Find 2×2 matrices A, B, C, $A \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ so that $AB = AC$ but $B \neq C$.
4. Find 2×2 matrices A, B so that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ but neither A nor B is the zero matrix.
5. If $CA = I_n$, A $m\times n$, C $n\times m$, show that A has n pivots. (Hint: show $A\vec{x} = \vec{0}$ has only triv. soln.)
6. (i) Show $(A + I_n)(A - I_n) = A^2 - I_n$ for A $n\times n$.
   (ii) Find 2×2 matrices A, B where $(A+B)(A-B) \neq A^2 - B^2$.
7. Suppose A is $n\times n$ and A^3 is the zero matrix. Show $A - I_n$ has an inverse (Hint: $\frac{1}{1-x} = 1 + x + x^2 + \cdots$)
8. Why can $A^{-1}B$ be computed by row reducing $[A \mid B]$? (Hint: $[A \mid B] \sim [I_n \mid A^{-1}B]$). When B is $n\times1$, how is this like solving $A\vec{x} = \vec{b}$?
9. Find a 2×3 matrix C where

$$C\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

   What is $\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} C$? Is C an inverse? (Is it I_3?)
10. Find a 3×2 matrix D where

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} D = I_2. \quad \text{Can } D\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} = I_3 ?$$

Examples

	injective (one-to-one)	not injective
surjective (onto)	$\vec{x} \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{x}$	$\vec{x} \mapsto \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vec{x}$
not surjective	$\vec{x} \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{x}$	$\vec{x} \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \vec{x}$