Discussion – Sep 12

1. Let A be 3×3 such that $T(\vec{x}) = A\vec{x}$ is surjective (maps $\mathbb{R}^3$ onto $\mathbb{R}^3$). Is T injective (one-to-one)? Now, if T is injective, is it surjective?
2. Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\vec{u} \in \mathbb{R}^n$ and $\vec{w} \in \mathbb{R}^m$ be a pair with $T(\vec{u}) = \vec{w}$. Show that $T(-\vec{u}) = -\vec{w}$.
3. Show that $T(\vec{0}) = \vec{0}$.
4. Suppose $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^3$ are points on a line in $\mathbb{R}^3$ (not necessary passing through the origin). Show $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a dependent set.
5. When are the following vectors independent?

$$\begin{bmatrix} a \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} b \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} c \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

6. Write the reduced form of a 3×3 matrix A such that the first two columns are pivot columns and

$$A\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

7. Find a matrix $R = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ with $a^2 + b^2 = 1$ so that

$$R\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$$ (R is a <u>Givens rotation</u>)

8. Multiply $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. What do all of these matrices represent geometrically?
9. Multiply (i) $\begin{pmatrix} 1 & 2 & 3 \\ & 1 & 2 \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ & 1 & 2 \\ & & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}(4\ 5\ 6)$

(iii) $\begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^k$ for $k = 1, 2, 3, \ldots$