Sep 7 — Disc 116

$$\left[\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$2R_1 - 3R_2 \longrightarrow R_2$

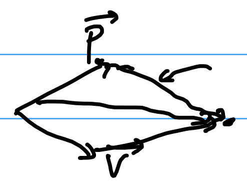
$2R_1 \rightarrow R_1$

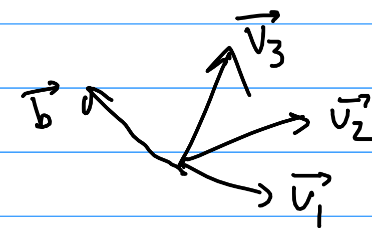
$R_2 \longleftrightarrow R_3$

$R_1 + 3R_4 \longrightarrow R_1$

---

24 c.





$\text{Span}\{\vec{v_1}, \vec{v_2}, \vec{v_3}\} = \mathbb{R}^2$

$\begin{bmatrix} \vec{v_1} & \vec{v_2} & \vec{v_3} \end{bmatrix} \vec{x} = \vec{b}$

16. $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \end{array}\right]$ $\qquad$ $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \end{array}\right]$

$x_1 = 2x_2 - 3x_3$ $\qquad$ $x_1 = 4 + 2x_2 - 3x_3$

x_2, x_3 free $\qquad$ x_2, x_3 free

$\vec{x} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ $\qquad$ $\vec{x} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$\vec{x} \in \text{Span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

20. line through $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ parallel to $\begin{bmatrix} -7 \\ 6 \end{bmatrix}$.

$\vec{x} = \underbrace{\begin{bmatrix} 3 \\ -2 \end{bmatrix}}_{\text{part?}} + \underbrace{t \begin{bmatrix} -7 \\ 6 \end{bmatrix}}_{\text{homog.?}}$

want:
$-7\vec{a_1} + 6\vec{a_2} = \vec{0}$

$\left[\begin{array}{cc|c} 6 & 7 & 0 \end{array}\right]$ works for homog.

$\left[\begin{array}{cc|c} 6 & 7 & 4 \end{array}\right]$ line is soln

(TFAE)

Theorem A is $m \times n$ matrix. The following are equivalent

a) For each $\vec{b} \in \mathbb{R}^m$ $A\vec{x} = \vec{b}$ has a solution.
i.e., $[A \mid \vec{b}]$ is consistent

b) Each $\vec{b} \in \mathbb{R}^m$ is a lin. comb. of columns of A.
$(A\vec{x})$

c) $\text{Span}\{\vec{a_1}, \dots, \vec{a_n}\} = \mathbb{R}^m$ ("columns of A span $\mathbb{R}^m$")

d) A has m pivots (A has a pivot in every row).

not a) There is a $\vec{b} \in \mathbb{R}^m$ with $[A \mid \vec{b}]$ inconsistent.

not b) There is a $\vec{b} \in \mathbb{R}^m$ which is not a lin. comb. of the columns of A.

not c) $\text{Span}\{\vec{a_1}, \dots, \vec{a_n}\} \subsetneq \mathbb{R}^m$ ("cols don't span")
↑ subset but not equal to

ex $\text{Span}\left\{\begin{bmatrix}1\\0\end{bmatrix}\right\} \subsetneq \mathbb{R}^2$

not d) There is a non-pivot row. (fewer than m pivots)
(rref (A) has an all-zero row)

ex $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$

$$T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$ map, function, transformation

$\mathbb{R}^n$ ↑ domain, $\mathbb{R}^m$ ↑ codomain

Matrix transformation : $T(\vec{x}) = A\vec{x}$ for some A

$\vec{x} \in \mathbb{R}^n$ so A has n columns

$A\vec{x} \in \mathbb{R}^m$, so A has m rows

A is $m \times n$.

for $\vec{x} \in \mathbb{R}^n$, $T(\vec{x})$ is an <u>image</u>

range or image of T

$T(\mathbb{R}^n) = \{T(\vec{x}) \mid \vec{x} \in \mathbb{R}^n\}$

image of T is $\mathbb{R}^m$ $\iff$ A pivot in every row.