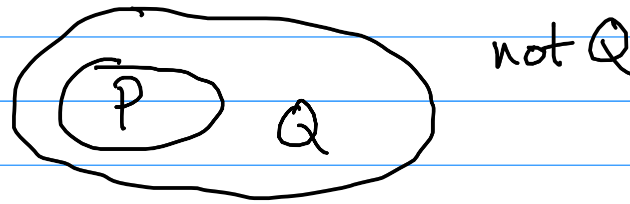
Sep 7 – Disc 115

$$\left[\begin{array}{cc|c} 1 & & 0 \\ & 1 & 0 \end{array}\right] \qquad \left[\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$*$ If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ lin. indep. set from $\mathbb{R}^4$

then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ also indep.

if P then Q



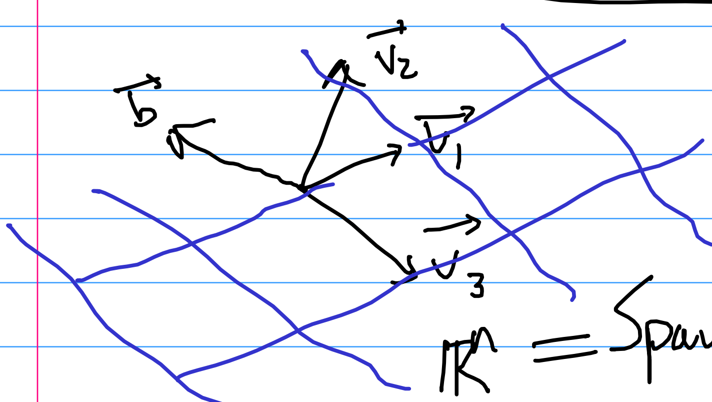
contrapositive : if not Q
then not P

contrapos. of $*$: if $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are dep., then $\{\vec{v}_1, \dots, \vec{v}_4\}$ are dep.

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$$

with c_1, c_2, c_3 not all zero.

then $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + 0\,\vec{v}_4 = \vec{0}$



2x3 matrix

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} \vec{x} = \vec{b}$$

$$\mathbb{R}^n = \text{Span}\{\vec{v}_1, \vec{v}_2\} \subset \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$$

"the following are equivalent"

Theorem If A is $m \times n$ matrix, TFAE

a) For each $\vec{b} \in \mathbb{R}^m$, $[A \mid \vec{b}]$ is consistent.
($A\vec{x} = \vec{b}$ has a solution)

b) Each $\vec{b} \in \mathbb{R}^m$ is a lin. comb. of columns of A.

c) $\text{Span}\{\vec{a}_1, \dots, \vec{a}_n\} = \mathbb{R}^m$
"the columns of A span $\mathbb{R}^m$"

d) pivot in every row.

not a) There is some $\vec{b} \in \mathbb{R}^m$ with $[A \mid \vec{b}]$ inconsistent.

not b) There is a $\vec{b} \in \mathbb{R}^m$ which is not a lin. comb. of cols of A.

not c) $\text{Span}\{\vec{a}_1, \dots, \vec{a}_n\} \subsetneq \mathbb{R}^m$

subset but not equal to

<u>not d)</u> there is not a pivot in every row

A is $m \times n$ with a pivot in every row.

$n \geq m$ is necessary

<u>ex</u> two vectors in $\mathbb{R}^3$. Do they span $\mathbb{R}^3$?

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

at most two pivots

<u>No</u> $\longleftarrow$ not a pivot in every row.

homogeneous system $A\vec{x} = \vec{0}$

always consistent ($\vec{x} = \vec{0}$ trivial solution)

existence of nontriv. soln
$\iff$ has free variable
(from row echelon form)

to show has non-triv. solution, sufficient to give one.

to show has <u>no</u> non-triv solutions
compute pivots with rref
to show no free vars.

parametric vector form.

$A \sim \mathrm{RREF}(A)$

$$\left(\begin{array}{ccc|c} 1 & 2 & & 0 \\ & & 1 & 0 \end{array}\right)$$

$x_1 = -2x_2$

x_2 free

$x_3 = 0$

$$\vec{x} = \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{array}\right)$$

$$\vec{x} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

$\vec{a}_1, \cdots, \vec{a}_n$ <u>lin. indep.</u> if

only solution to $[\vec{a}_1 \ \vec{a}_2 \ \cdots \ \vec{a}_n]\vec{x} = \vec{0}$

is trivial.

<u>dep.</u> if not indep.

lin. dep. rel. is the nontrivial soln.

<u>ex</u> $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$

dep. since: $\quad 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 1\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$