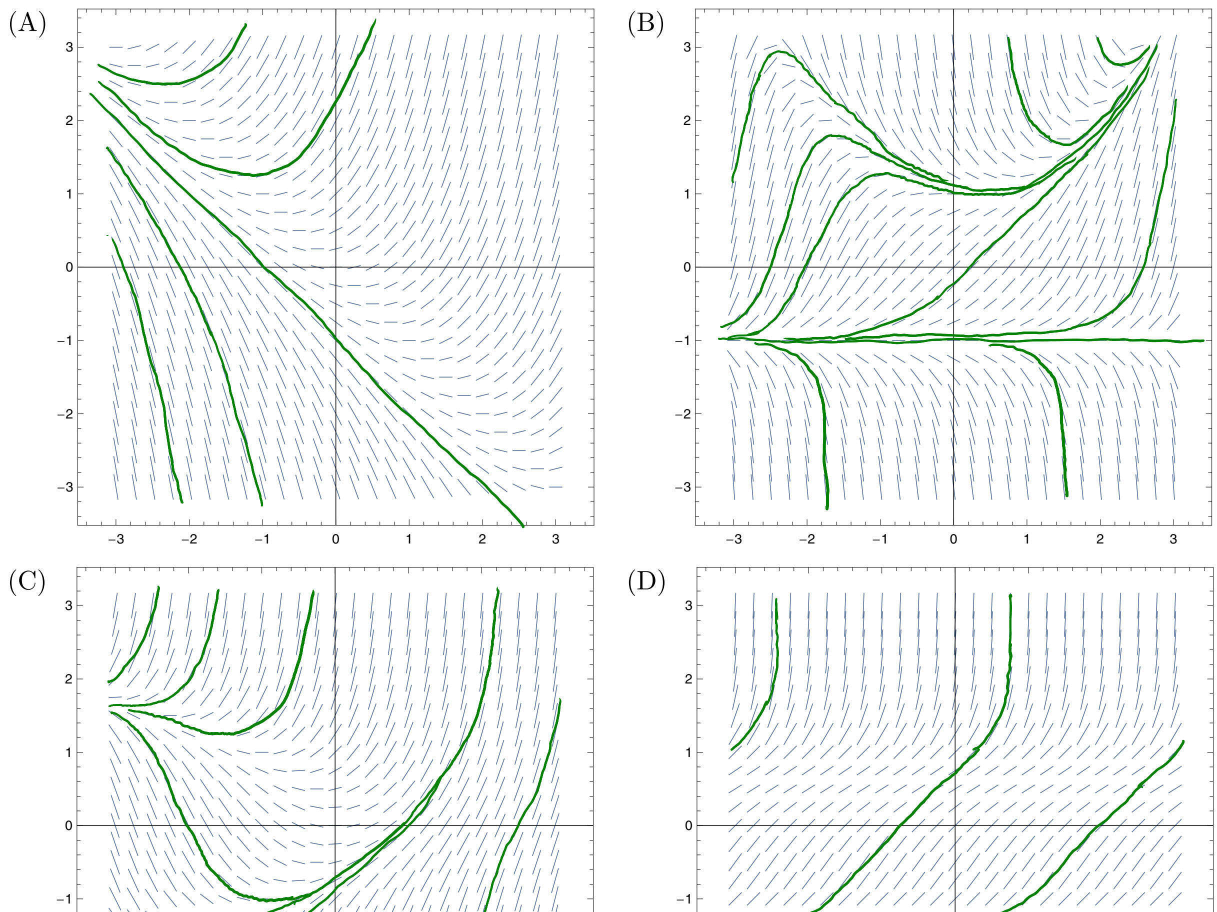
Math 1B: Calculus **Spring 2020**

Discussion 23: Direction Fields (9.2)

Instructor: Alexander Paulin Date: April 3, 2020

1. Consider the following four direction fields (also known as slope fields).



   (a) Draw some representative solution curves on these direction fields.

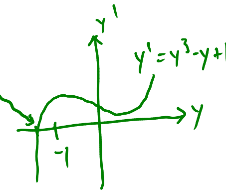
   (b) Which of these look autonomous[1]?

   Only (D) does — the slopes do not depend on x

[1] A direction field is *autonomous* if it corresponds to a differential equation of the form $y' = f(y)$ for some function f.

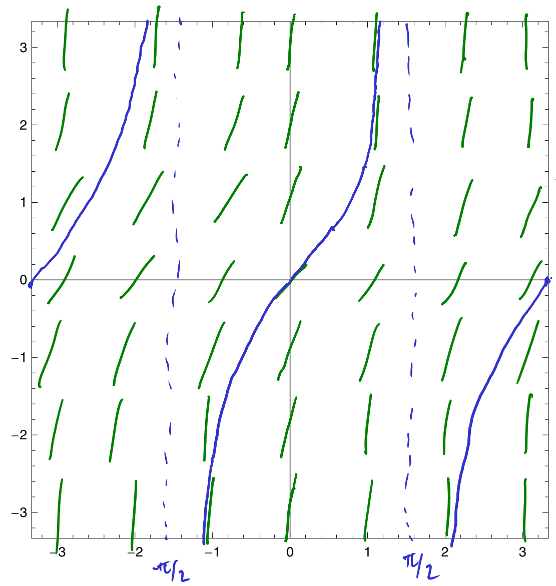
(c) For each of these differential equations, determine which direction field it corresponds to (if any):

		@ $(x,y)=(0,0)$	Autonomous?	equil. solns	@ $(x,y)=(-2,-2)$
(C)	i. $y' = x + y^2$	$y'=0$	N	none	$y' = 2 > 0$
(B)	ii. $y' = (y+1)(\frac{1}{3}x^2 + 1 - y)$	$y'=1$	N	$y=-1$	
(A)	iii. $y' = x + y$	$y'=0$	N	none	$y' = -4 < 0$
None	iv. $y' = y + 1$	$y'=1$	y	$y=-1$	
(D)	v. $y' = y^3 - y + 1$	$y'=1$	y	$y =$ • (something < 1)	



2. Consider the differential equation $y' = 1 + y^2$.

   (a) Carefully draw a direction field:



   (b) Verify that $y = \tan(x)$ is a solution.

   $y' = \sec^2(x) = 1 + \tan^2(x) = 1 + y^2$

   (c) Plot $y = \tan(x)$ over your direction field.

   (d) Take a moment to contemplate the remarkable fact that $y = \tan(x)$ has vertical asymptotes, yet the direction field has no discontinuities! wow.

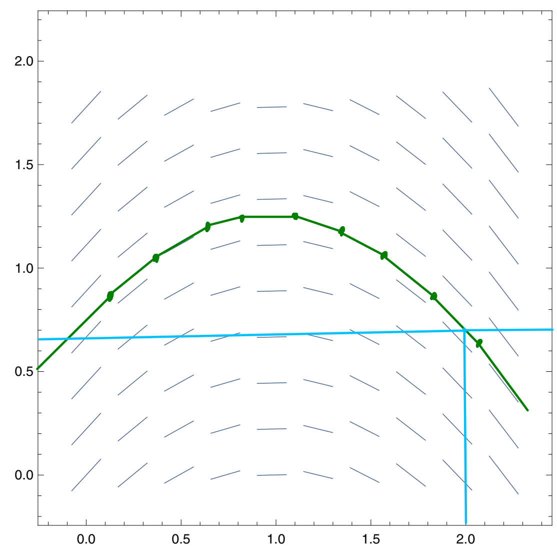
   (e) For $C \in \mathbb{R}$, why is $y = \tan(x - C)$ also a solution?[2]

   $y' = 1 + y^2$ is autonomous, so horizontal shifts of a solution are still solutions.

   (Direction fields of an autonomous differential eqn. are "horizontal-shift invariant")

[2] *Hint: what is the relationship between the graphs of $y = f(x)$ and $y = f(x - C)$ in general?*

3. Consider the following direction field for a differential equation:



(a) For the initial value problem $f(0) = 0.5$, draw a approximation of a solution curve in the following manner. Your approximate solution curve should consist of many short line segments, and each segment should be parallel to some nearby line segment from the direction field.

(b) Given your approximate solution curve, what is your estimate for $f(2)$?

$f(2) \approx 0.65$

(c) Do you think this is an overestimate or an underestimate for the true value of $f(2)$? Why?

Overestimate: solutions would be concave down, and line segments systematically go above curve they are tangent to.

(d) This direction field is from the differential equation $y' = 1 - x$. Solve the above initial value problem and calculate the exact value of $f(2)$.

$y = \int (1-x)\,dx = x - \frac{1}{2}x^2 + C$

$\frac{1}{2} = f(0) = 0 - \frac{1}{2}0^2 + C$, so $C = \frac{1}{2}$

$f(2) = 2 - \frac{1}{2}2^2 + \frac{1}{2} = 2 - 2 + \frac{1}{2} = \boxed{\frac{1}{2}}$

So, indeed, 0.65 was an overestimate.