Math 1B: Calculus **(Spring 2020)**

Discussion 28: Second-Order Homogeneous Linear Equations

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- Theorem 1: If $y_1(x)$ and $y_2(x)$ are both solutions of the linear homogeneous equation

$$P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)y = G(x), \qquad (1)$$

$where\ P, Q, R, and\ G\ are\ continuous\ functions, then\ the\ function$

$$y(x) = c_1y_1(x) + c_2y_2(x)$$

is also a solution.

- Theorem 2: If $y_1(x)$ and $y_2(x)$ are linearly independent solutions of (1), where P(x) is never 0, then the general solution is given by

$$y(x) = c_1y_1(x) + c_2y_2(x)$$

- General solution has 3 cases based on the discriminant
  - Case 1: $b^2 - 4ac > 0$ (real and distinct) → $y = c_1e^{r_1x} + c_2e^{r_2x}$
  - Case 2: $b^2 - 4ac = 0$ (real and equal) → $y = c_1e^{rx} + c_2xe^{rx}$
  - Case 3: $b^2 - 4ac < 0$ (complex numbers) → $y = e^{ax}(c_1\cos(Bx) + c_2\sin(Bx))$

1. Find the general solution to the following differential equations:
   a. $y" + y' - 6y = 0$

   b. $3\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

   c. $4y" + 12y' + 9y = 0$

   d. $y" - 6y' + 13y = 0$

2. Solve the following initial value problems:
   a. $y" + y' - 6y = 0, \quad y(0) = 1, \; y'(0) = 0$

   b. $y" + y = 0, \quad y(0) = 2, \; y'(0) = 3$

3. Solve the following boundary value problem:
   $y" + 2y' + y = 0, \quad y(0) = 1, y'(1) = 3$