Math 1B: Calculus	**Spring 2020**

Discussion 18: Taylor and Maclaurin Series

Maclaurin Series

1. $f(x) = \sin x$

f(x) = sin x	f(0) = 0	Maclaurin series: For all x
f '(x) = cos x	f '(0) = 1	$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
f "(x) = - sin x	f "(0) = 0	$\left\| f^{n+1}(x) \right\| \le 1$
f '"(x) = - cos x	f '"(0) = -1	

Taylor's inequality: M = 1

$\left| R_n(x) \right| \le \frac{\left| x^{n+1} \right|}{(M+1)!}$

$n \to \infty, \left| R_n(x) \right| \to 0$ Therefore, sin x is the sum of its Macluarin series for all x.

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2. $f(x) = 2^x$

f(x) = 2^x	f(0) = 1	Maclaurin series:
f '(x) = $(\ln 2)2^x$	f '(0) = ln2	$f(x) = 1 + \frac{\ln 2}{1!} x + \frac{(\ln 2)^2}{2!} x^2 + \frac{(\ln 2)^3}{3!} x^3 + ... = \sum_{n=0}^{\infty} (\ln 2)^n \frac{x^n}{n!}$
f "(x) = $(\ln 2)^2 2^x$	f "(0) = $(\ln 2)^2$	
f '"(x) = $(\ln 2)^3 2^x$	f '"(0) = -1	

Ratio test: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 < 1$, for all x. $R = \infty$

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3. Evaluate

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$

We know $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right) - 1 - x}{x^2}$$

$$= \lim_{x\to 0} \frac{\left(\frac{x^2}{2!} + \frac{x^3}{3!} + ...\right)}{x^2} = \lim_{x\to 0} \frac{1}{2} + \frac{x}{3!} + \frac{x^2}{4!} + ... = \frac{1}{2}$$

Taylor Series

1. $f(x)=x^6-x^4+2, a=-2$

$f(x)=x^6-x^4+2$	$f(-2)=50$	Taylor series: $f(x)=50+(-160)(x+2)+\frac{432}{2!}(x+2)^2+\frac{-912}{3!}(x+2)^3+....$
$f'(x)=6x^5-4x^3$	$f'(-2)=-160$	
$f''(x)=30x^4-12x^2$	$f''(-2)=432$	
$f'''(x)=120x^3-24x$	$f'''(-2)=-912$	

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2. $f(x)=\sqrt{x}, a=16$

$f(x)=\sqrt{x}$	$f(16)=4$
$f'(x)=\frac{1}{2x^{\frac{1}{2}}}$	$f'(16)=\frac{1}{2.4}$
$f''(x)=-\frac{1}{2.2x^{\frac{3}{2}}}$	$f''(16)=-\frac{1}{2.2.4^3}$
$f'''(x)=\frac{3}{2.2.2x^{\frac{5}{2}}}$	$f'''(16)=\frac{3}{2.2.2.4^5}$
$f^4(x)=\frac{3.5}{2.2.2.2x^{\frac{7}{2}}}$	$f^4(16)=-\frac{3.5}{2.2.2.2.4^7}$

Taylor series:

$$f(x)=4+\frac{1}{2.4}(x-16)-\frac{1}{2.2.4^3}(x-16)^2+\frac{3}{2.2.2.4^5}(x-16)^3-\frac{3.5}{2.2.2.2.4^7}(x-16)^4+.....$$
$$=4+\frac{(x-16)}{8}+\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1.3.5....(2n-3)}{2^{5n-2}.n!}(x-16)^n$$

Ratio test: , for all x.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{x}{16}-1\right|<1 \Rightarrow converging$$
$0<x<32$

$R=16$