Math 1B

Power series

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1. For what values of x is the series $\sum_{n=1}^{\infty} n^n x^n$ convergent or divergent?

2. Find the radius of convergence and interval of convergence:

(a) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$

(b) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n-1}$

(c) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n\, 5^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}$

<u>Theorem</u> For $\sum_{n=0}^{\infty} c_n (x-a)^n$, these are the possibilities for the interval of convergence:

1) $\{a\}$ (radius 0)
2) $(-\infty,\infty)$ (radius ∞)
3) $(a-R, a+R)$, $[a-R, a+R)$, $(a-R, a+R]$, $[a-R, a+R]$ (radius R)

<u>Theorem</u> A power series absolutely converges in the interior of its interval of convergence.

<u>Theorem</u> If $\lim_{n\to\infty} |c_n|^{1/n} = c \neq 0$, then the radius of convergence of $\sum_{n=0}^{\infty} c_n x^n$ is $1/c$.

<u>Theorem</u> If $\lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = R$ exists, then the radius of convergence of $\sum_{n=0}^{\infty} c_n (x-a)^n$ is R.