Math 1B

Alternating series

March 4
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1. Do the following series converge or diverge?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5n+3}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5n-3}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$

(f) $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$

(g) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

(h) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+3}$

2. How many terms do you need to calculate $\ln(2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ to within 0.001 ?

3. (a) For what values of p does $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$ converge?

(b) How many terms do you need to calculate it to within 0.001?

Alternating series test

If $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ has $b_n \geq b_{n+1} \geq 0$ for all n and $\lim_{n\to\infty} b_n = 0$,

then the series is convergent

Alternating series estimation

If $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ has $b_n \geq b_{n+1} \geq 0$ for all n and $\lim_{n\to\infty} b_n = 0$,

then $\left| \sum_{n=1}^{\infty} (-1)^{n-1} b_n - \sum_{n=1}^{N} (-1)^{n-1} b_n \right| \leq b_{N+1}$