Math 1B

Comparison tests

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1. Do the following series converge or diverge?

(a) $\sum_{n=0}^{\infty} \frac{1}{n^2+6}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2-6}$

(c) $\sum_{n=1}^{\infty} \frac{3}{n-\frac{2}{3}}$

(d) $\sum_{n=1}^{\infty} \frac{1}{6n+27}$

(e) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$

(f) $\sum_{n=1}^{\infty} \frac{n+1}{n^3+n+1}$

2. If $\sum_{n=1}^{\infty} a_n$ is convergent with $a_n > 0$ for all n,

(a) Must $\sum_{n=1}^{\infty} a_n^2$ converge?

(b) Must $\sum_{n=1}^{\infty} \sqrt{a_n}$ converge?

Comparison test Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms, $0 \le a_n \le b_n$ for all n.

- If $\sum a_n$ diverges, then $\sum b_n$ diverges.
- If $\sum b_n$ converges, then $\sum a_n$ converges. (contrapositive)

Limit comparison test Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$ exists and $c > 0$, (and not ∞)

then $\sum a_n$ converges if and only if $\sum b_n$ converges.