

I. Comparison theorems

Determine whether each integral is convergent or divergent.

$$1. \int_1^\infty \frac{x}{x^3+1} dx$$

$$2. \int_1^\infty \frac{1 + \sin^2(x)}{\sqrt{x}} dx$$

$$3. \int_0^1 \frac{\sec^2(x)}{x\sqrt{x}} dx$$

II. Arc length

Find the lengths of the described curves.

$$\text{Recall: } \frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

1. $36y^2 = (x^2 - 4)^3, \quad 2 \leq x \leq 3, \quad y \geq 0 \quad (\text{Sketch the curve})$

2. $x = \frac{1}{3}\sqrt{y}(y-3), \quad 1 \leq y \leq 9$

3. $y = \ln(\sec(x)), \quad 0 \leq x \leq \frac{\pi}{4}$

4. $y = \sqrt{x-x^2} + \arcsin(\sqrt{x})$