

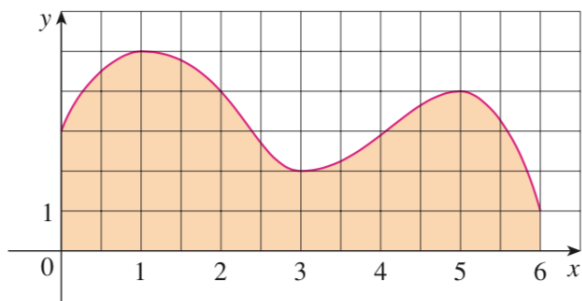
## Discussion 6: Approximate Integration

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**1 Approximate using Midpoint/Trapezoid/Simpson's Rule:**

1. Estimate the area under the graph in the figure by using (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule, each with  $n = 6$ .



2. Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with  $n = 8$ :

$$\int_0^2 \sqrt{1+x^2} dx$$

**2 Given error bound, calculate n or vice-versa:**

1. (a) Find the approximations  $T_s$  and  $M_s$  for the integral  $\int_0^1 \cos(x^2)dx$ .  
  
(b) Estimate the errors in the approximations of part (a).  
  
(c) How large do we have to choose  $n$  so that the approximations  $T_n$  and  $M_n$  to the integral in part (a) are accurate to within 0.0001?
  
2. (a) Find the approximations of  $T_{10}$ ,  $M_{10}$ , and  $S_{10}$  for  $\int_0^\pi \sin x dx$  and the corresponding errors  $E_T$ ,  $E_M$ , and  $E_S$ .  
  
(b) How large do we have to choose  $n$  so that the approximations  $T_n$ ,  $M_n$ , and  $S_n$  to the integral in part (a) are accurate to within 0.00001?
  
3. How large should  $n$  be to guarantee that the Simpson's Rule approximation to  $\int_0^1 e^{x^2} dx$  is accurate to within 0.00001?