

Background

| Expression         | Trig. subst.  | Hyp. trig. subst  |
|--------------------|---|---|
| $\sqrt{a^2 - x^2}$ | $x = a \sin \theta$ ( $\cos^2 \theta = 1 - \sin^2 \theta$ ) | $x = a \tanh t$ ( $\operatorname{sech}^2 t = 1 - \tanh^2 t$ ) |
| $\sqrt{a^2 + x^2}$ | $x = a \tan \theta$ ( $\sec^2 \theta = 1 + \tan^2 \theta$ ) | $x = a \sinh t$ ( $\cosh^2 t = 1 + \sinh^2 t$ )               |
| $\sqrt{x^2 - a^2}$ | $x = a \sec \theta$ ( $\tan^2 \theta = \sec^2 \theta - 1$ ) | $x = a \cosh t$ ( $\sinh^2 t = \cosh^2 t - 1$ )               |

Warmup

- 1)  $\cos(\tan^{-1}(\frac{x}{a})) =$
- 2)  $\sin(\sec^{-1}(\frac{1}{\sqrt{a^2+x^2}})) =$
- 3)  $\sin(\cos^{-1}(x)) =$
- 4) Find the area of a circle of radius  $R$ . (Hint:  $x^2 + y^2 = R^2$ )

Integrals

1.  $\int \frac{1}{(x^2+2x+5)^{3/2}} dx$       and       $\int_{-1}^1 \frac{1}{(x^2+2x+5)^{3/2}} dx$

2.  $\int \frac{dx}{\sqrt{x^2-9}}$

$$3. \int \frac{1}{(x^2+16)^{1/2}} dx$$

$$4. \int \frac{\sqrt{x^2-9}}{x^3} dx$$

$$5. \int x^2 (a^2 - x^2)^{1/2} dx$$

$$6. \int \frac{x}{\sqrt{x^2+4}} dx \text{ by (a) u-sub. (b) trig. sub.}$$