Math 1B: Calculus **(Spring 2020)**

Discussion 24: Models for Population Growth

Instructor: Alexander Paulin Date April 8, 2020,

- Recall equation for relative growth: $\frac{dP}{dt} = kP, \quad P(0) = P_0, \quad P(t) = P_0 e^{kt}$
- To account for emigration (or "harvesting") at constant M: $\frac{dP}{dt} = kP - M$
- To account for carrying capacity: $\frac{dP}{dt} = kP(1 - \frac{P}{M})$ (Logistic Differential Equation)

1. Derive the solution for the logistic differential equation. (Note: It is a Separable Diff EQ)

2. For the following $\frac{dP}{dt} = 0.4P - 0.001P^2, \qquad P(0) = 50$
   a. What is the carrying capacity?
   b. What is P’(0)?
   c. When will the population reach 50% of the carrying capacity?

3. The population of the world was about 6.1 billion in 2000. Birth rates around that time ranged from 35 to 40 million per year and death rates ranged from 15 to 20 million per year. Let's assume that the carrying capacity for world population is 20 billion.

(a) Write the logistic differential equation for these data. (Because the initial population is small compared to the carrying capacity, you can take k to be an estimate of the initial relative growth rate.)

(b) Use the logistic model to estimate the world population in the year 2010 and compare with the actual population of 6.9 billion.

(c) Use the logistic model to predict the world population in the years 2100 and 2500.

4. Biologists stocked a lake with 400 fish and estimated the carrying capacity (the maximal population for the fish of that species in that lake) to be 10,000. The number of fish tripled in the first year.

   (a) Assuming that the size of the fish population satisfies the logistic equation, find an expression for the size of the population after t years.

   (b) How long will it take for the population to increase to 5000?